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RESULTS OF RECENT CEMENT RESEARCH AND THEIR PARTIAL APPLICATION.*

BY

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A great deal has been learned in the last few years about the composition and hydration of portland cement and a great deal has also been learned about the manufacture of concrete from the cement. But there still remains much to be learned. The present paper may serve to indicate not only some of the knowledge that has been gained, but also some of the knowledge and experience that are lacking. Firstly, there will be described herein some of the more important results of recent cement research, including a description of the process by which strength is developed. Next, an attempt will be made to show how properties of concrete can be partially controlled by various means. Finally, there will be discussed some problems encountered in important structures showing the relation of cement research to the solution of those problems. It should become clear that research results cannot often be applied in full because questions of cost and experience must always be considered.

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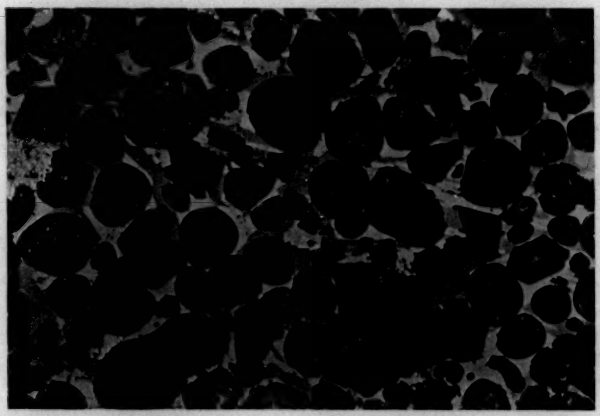
COMPOSITION OF CEMENT.

In 1929 Dr. R. H. Bogue published a paper that removed much of the mystery surrounding portland cement. The paper was entitled "Calculation of Compounds in Portland Cement." It gave chemists and engineers an opportunity to translate the usual chemical analysis into something that has a close relation with the behavior of the cement.

Dr. Bogue's work told chemists and engineers that cement is composed principally of four major compounds, and pro-

FIG. 1.



Polished section of low heat clinker. Composition: 3CS 22 per cent., 2CS 60 per cent., 3CA 5 per cent., 4CAF 15 per cent. Magnification: 500 times.

vided a method of computing the amounts of each present. The compounds are called tricalcium silicate, dicalcium silicate, tricalcium aluminate, and tetracalcium aluminoferrite. For convenience the symbols 3CS, 2CS, 3CA, and 4CAF respectively, are used to designate the compounds.

The four compounds are well illustrated in Fig. 1, which is a microphotograph of a polished section of actual cement clinker, prepared by a clever process adapted to this purpose by Dr. Herbert Insley of the National Bureau of Standards.

A piece of cement clinker was ground plane, then polished with rouge and finally etched with a weak solution of nitric acid in alcohol for 5 seconds. The acid corroded each mineral differently and thus produced a contrast that permits individual minerals to be seen plainly. The round black crystals are 2CS. The hexagonal, dark crystals are 3CS. The white interstitial material is 4CAF. The grey interstitial material is uncrystallized glass and contains the 3CA. This happens to be the clinker of a low-heat cement in which the content of dicalcium silicate is unusually high and there is less than the usual amount of both 3CS and 3CA.

CEMENTING ABILITY OF CEMENT COMPOUNDS.

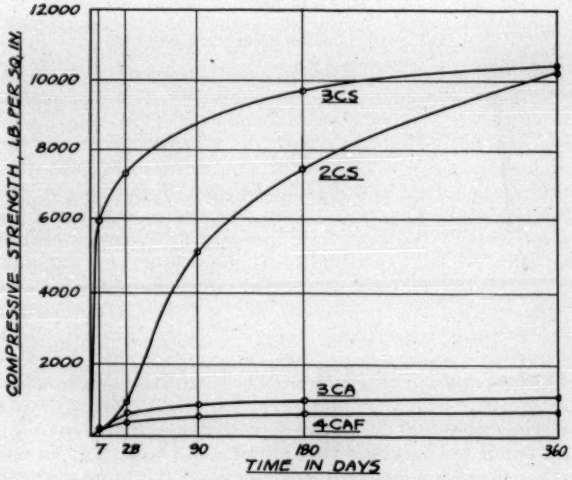
As soon as Bogue's work made the compounds in portland cement generally known, investigators began to inquire into the properties of each of the compounds to learn which were desirable and which were not. Perhaps strangely, it was found that all of the major compounds in cement, and most of the minor compounds, react with water. It soon developed that the silicates, 3CS and 2CS, are the only compounds, however, that are capable of developing great strength upon hydration. They are therefore termed the cementing minerals. The compounds containing iron and alumina are detrimental, or at best, adulterating minerals. The two silicates resemble one another except for the speed with which they work. Tricalcium silicate is active and under ordinary conditions reacts with water and hardens many times as fast as the 2CS. Thus when pulverized 3CS is mixed with water to form a paste and cast into a form and stored at room temperature, it will develop a strength of about 6,000 p.s.i. in one week. Correspondingly, 2CS will develop the same strength in about 3 or 4 months, but will be scarcely hard enough to handle in one week.

The difference in rate of hydration of the two calcium silicates is probably much greater than is indicated by comparing the time required to develop equal strengths. When a specimen of 2CS that has developed a strength of 6,000 p.s.i. is examined, it is found to be only fractionally hydrated. On the other hand, the corresponding 3CS has been largely converted into hydration products. Thus the potential

strength producing power of 2CS is not realized and the attempt to utilize this compound more fully is a promising subject for research.

The compounds containing iron and alumina develop some strength after being mixed with water but their strength does not compare with that of the silicates. Tests on cements containing varying amounts of these minerals show that increased amounts of them, particularly the 3CA, reduce the long-time strength of portland cement mixtures.

FIG. 2.



Comparison of strength developed by individual compounds of cement.

The relative strength-producing properties of the individual compounds may be visualized by referring to Fig. 2, which shows the strength of each compound at various ages after being mixed with water.

PROCESS OF STRENGTH DEVELOPMENT.

Before considering other properties of concrete as affected by the composition of cement, it may be well to picture the

process of strength development. Since the two calcium silicate compounds are the only ones that are capable of developing much strength, the picture can be simplified by considering their hydration separately. The hydration of the 2CS is believed to be relatively simple. When granulated 2CS is mixed with water, nothing appears to happen for many hours. But something does happen. The water removes the molecules one by one from the surface of each grain and disperses them. Thus the compact molecules are isolated and surrounded by water and are built up in bulky form on the surface of each grain. As more and more of the 2CS is removed from the compact grains, more and more water finds a molecule to cling to and a gradual drying action results. There may be no evaporation from the mixture but as the 2CS molecules are removed from the grains, a tremendous amount of surface is provided on which the water may attach itself and free water becomes scarce. The bulky, dispersed 2CS that the water has produced by separating the individual molecules is called gel and this gel gradually fills practically all of the interparticle spaces. When a grain of 2CS is changed to gel, the volume of the gel may be several times that of the unhydrated grain.

The manner in which the gel becomes denser and stronger can be visualized by resorting again to an analogy. Since the molecules of 2CS are far too small to be seen even with a microscope, suppose balls of dry clay are allowed to represent the grains of 2CS, so that the molecules will be represented by the clay particles. Place a few such balls of clay in a bowl and cover them with water. That represents to enlarged scale the fresh paste of dicalcium silicate. Now to speed up the simulated hydration, stir the balls of clay around in the water. The surface particles of the clay balls disperse through the water, which soon becomes a slurry. The slurry may represent the first stage in the development of the gel. As the stirring is continued, more and more of the clay is dispersed from the balls and the slurry gets thicker. Finally the mixture is more or less solidified but the core of each clay ball is still dry. In this advanced stage, the particles of clay distributed through the water are much more closely packed than they were when the slurry was first formed.

This corresponds to the hardening of the gel. The further dispersion of dry clay from the interior of the clay balls is now difficult because the gel-like moist clay around each core makes it difficult for water to reach the dry clay. This simulates the slowing down of hydration.

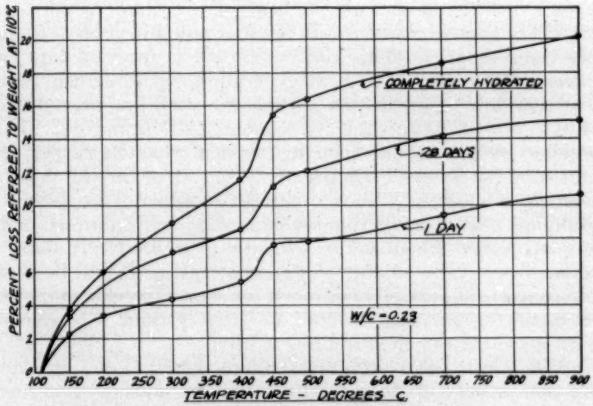
There must be recognized one essential difference between the clay slurry, in which there are clay particles surrounded by water, and the dicalcium silicate gel, in which there are molecules surrounded by water. In the case of the gel, when the molecules get close enough together, their corners connect definitely and the gel then becomes a skeleton of molecules fastened together at miscellaneous points. The water can then be removed by evaporation or other means, leaving the minutely-porous but rigid gel that fills most of the inter-particle spaces in hardened concrete. When the water is thus removed, more connections between the silicate molecules are permitted, the mixture shrinks slightly, and when saturated again does not expand to its original condition. However, concrete and other cement mixtures do not shrink unless they are dried.

The hydration of 3CS is different mainly in that it is faster and that it produces a byproduct. Since each 3CS molecule has an extra unit of lime, the compound is less stable and the water can pull the molecules out faster. The same sort of gel is produced as from 2CS. The extra lime does not become a part of the gel but collects and crystallizes as calcium hydroxide wherever space is available. Hydration has been speeded up through the instability caused by the content of extra lime, which must be discarded. Thus 3CS does not produce 100 per cent. gel and in this respect is inferior to the 2CS. But the 2CS is so slow in hardening as to be unable to capitalize on its gel producing ability.

If a sample of hydrated cement is dried in an oven at 110° C. and then heated to higher temperatures, the losses in weight give some indication of the nature of the hydration products. The water does not leave the gel readily and high temperature or high vacuum is necessary to remove all of it. The open spaces in the gel are of varying size and can be likened to capillary tubes, the smaller the tube, the more difficult it is to evaporate water contained in it. Thus an

appreciable amount of water remains in the gel even above the boiling point of free water. This makes it evident that the pores in the gel are extremely small. There is no hope of seeing them with any microscope. Figure 3, taken from the work of F. M. Lea of the Building Research Station in England, shows how the moisture is gradually driven from hydrated cement as the temperature is raised. The jump in the curves at about 425° C. indicates dehydration of the

FIG. 3.



Loss of moisture from hydrated cement at different temperatures.

calcium hydroxide, which was formed as a byproduct during hydration.

OTHER PROPERTIES OF COMPOUNDS.

Consider now some other properties of concrete as affected by composition of cement. Usually, strength is so far in excess of requirements that it serves mainly as an index of the general quality of the concrete. In massive structures, where heat cannot escape readily, the heat generated by cement upon hydration is important. In fact, the most troublesome feature of concrete for dams comes from the tendency to rise in temperature as heat is liberated by the

cement. The trouble is not caused by the rise in temperature, which occurs while the concrete is relatively soft and yielding, but by the later cooling and tendency to contract when the concrete is rigid and more or less fixed in position. The temperature rise and fall necessitates the use of costly contraction joints and grouting systems, and in some cases artificial refrigeration, to partially nullify its effects.

The compounds in portland cement have almost the same order of desirability for heat generation as they had for strength. One per cent. of 3CA liberates twice as much heat upon hydration as does one per cent. of 3CS, and one per cent. of 3CS in turn liberates about twice as much heat as does one per cent. of 2CS. The iron compound is intermediate between 3CS and 2CS in heat generation. The cement with very low heat of hydration will be one with low percentages of 3CA and 3CS, with the 2CS content predominating. This will be a slow hardening cement. Such a cement was specified and used in Boulder Dam after being tried in the Morris Dam in California. Such a cement would not be best for buildings composed of thin walls, beams, and columns, or in any structure where early strength is desired. If such a cement were used for thin work, construction should be slow, the concrete should be kept moist for several weeks, and even then, drying shrinkage would be greater than if a cement with more tricalcium silicate were used.

While in massive structures volume changes and cracking are due mainly to temperature changes, in thin structures similar troubles come from drying shrinkage. The composition of cement is not the most important factor in drying shrinkage but still it has some effect. Again, almost the same order of desirability of individual compounds in cement prevails. Tests on concretes in which the pure compounds are used in place of cement show that lowest drying shrinkage is exhibited by the concrete containing 3CS. Greatest shrinkage is exhibited by the concrete containing 3CA, the same compound that was most detrimental to strength and that contributed most to heat liberation. Concrete containing pure 2CS shrinks somewhat more than that containing pure 3CS, but the difference is not great if the period of moist curing is long.

Another important property of concrete is the durability. Two actions are usually considered, that of alternate freezing and thawing and that of corrosive waters such as sodium sulfate. The resistance of a concrete to freezing and thawing is so closely controlled by strength and density that it is difficult to say that one compound is better than another except as it improves the strength. Thus, under ordinary conditions, 3CS is again the most desirable compound and 3CA is the least desirable as far as resistance to freezing and thawing is concerned.

The resistance of concrete to the disintegrating action of sodium sulfate is quite definitely related to the composition of the cement. The silicates are excellent in resisting sodium sulfate with perhaps a little in favor of the 3CS again. The alumina and iron bearing compounds are both detrimental, but tricalcium aluminate is so much the worse of the two that the resistance to sodium sulfate of a hardened neat cement paste is almost a sure index of the amount of 3CA present.

There are other properties of concrete for the testing engineer to worry about, but if introduced here, they will only confuse the picture.

WHY THE PRESENT COMPOSITION OF CEMENT.

With so much in favor of the silicates in cement and so much against the 3CA, why is not cement made only of the silicates? The answer is, first, impracticability, and second, lack of necessity. The requirements for most concrete do not justify the great expense of such a cement. Standard portland cement is a good product and one of its splendid features is that limestone and clay from which it can be made are distributed all through the world. To make the cement entirely of silicates would require pure limestone, which is fairly common, and pure silica, which is also fairly common, but not usually adjacent to the pure limestone. Obtaining the materials is simple compared to making the calcium silicates from them. The iron and alumina contained in clay acts as a flux in the burning of cement clinker and without them higher temperatures and greatly refined control would be necessary. At present the calcium silicates make up about three-quarters of the average cement. To make the other

quarter also of calcium silicates instead of the iron and alumina compounds would multiply the cost several times. A compromise can be made, however, without great increase in cost by mills that are favored with suitable raw materials and this is exactly what was done for a portion of the Colorado River Aqueduct in California. The problem there was to obtain a cement that would resist the sulfate soils through which part of the distribution system of the aqueduct passed. Concrete containing ordinary cement disintegrated in a few months but tests showed that the special cement made concrete that was able to resist the sodium sulfate. A limit was placed on the combined percentage of 3CA and 4CAF. The slight extra cost of the superior cement was well warranted in such a case.

FINENESS OF CEMENT.

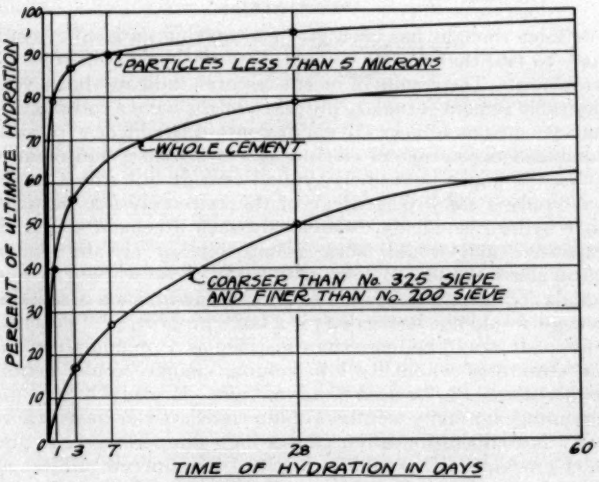
The optimum fineness of cement has been a subject of controversy for many years. There is no question but that finer cements exhibit certain advantages and that under some conditions of use they exhibit disadvantages. Whatever setting and hardening reactions occur in a coarse cement, occur more rapidly in a finer cement. Concretes made with finer cements are generally stronger, less permeable, and finer textured. But if made with the same cement content and aggregate gradation, they are also more brittle and liberate heat more rapidly at the early ages. Thus under some conditions cracking of concrete may be accentuated by substituting a finer cement for a coarser one. Contrary to common belief, the drying shrinkage of concrete is not affected greatly by the changes in fineness of cement that are usually encountered.

Some compositions of cement are not suited to high fineness and may give quickest troubles as well as sudden temperature rise in structures when ground too finely. Cements high in 3CA content give most trouble in this respect. If a cement were composed almost entirely of 2CS, the finer it were ground the better it would be from practically every standpoint. Even if it were ground to double the fineness of an average cement, it would not generate heat at a troublesome rate and probably would not give quickset troubles.

But 2CS is difficult to grind and cements of extremely high 2CS content await a vast improvement in grinding efficiency before they can be economically ground to high fineness.

Until recent years, standard portland cements were relatively coarse and contained fairly high percentages of dicalcium silicate. Tests on those cements led us to believe that coarse particles of cement do not hydrate appreciably and that only a small percentage of a cement hydrates in

FIG. 4.



Effect of particle size on rate of hydration of cement.

ordinary concrete. Recent research indicates that modern cements are largely transformed into hydration products. Figure 4 shows the rate of hydration of a modern cement and of air-separated fractions of the same cement to show the effect of particle size. The degree of hydration was determined for each point on the curves in Fig. 4 by comparing the heat of hydration of a partially hydrated sample with that of a fully hydrated sample. The latter sample was made by

repeatedly grinding a specimen of the cement, remixing it with water and allowing it to harden, until practically no unhydrated cement remained. The heats of hydration were measured by the heat-of-solution method.

Referring to Fig. 4, it may be seen that the finest particles are 90 per cent. hydrated in a week and that the whole cement is about 80 per cent. hydrated in a month. The particles coarser than a No. 325 sieve hydrate much slower, but even these particles are about two-thirds hydrated in two months time.

IDEAL CEMENT.

Some thought has been given to making an ideal cement, only to find that no one cement, however good, will solve all problems. The results of recent research indicate that a very desirable cement for many purposes might have a composition and fineness as follows: It might contain the highest possible combined percentage of calcium silicates with a minimum of 3CA. It might be a finely ground cement but still it might have only a small percentage of the extremely fine particles that hydrate suddenly. Obviously such a cement would be expensive and would work a hardship on certain plants, particularly the less modern plants and those whose raw materials contain much alumina. Furthermore, such a superior cement would not be needed for a large proportion of concrete work. It would be more than justified as a premium cement for structures where its low volume change would permit higher stresses to be used to advantage. It would have some advantage for many members of intermediate thickness, where early heat liberation often causes high temperature rise and later cracking. It would be justified for concrete subject to corrosive waters. But it would not solve the problem of cracking in very thin structures. Nor would it solve a variety of problems that are encountered in special cases.

CONTROL OF PROPERTIES.

Control of composition and fineness of cement will not solve all problems because properties of concrete are affected also by changes in other materials and conditions. Since the scope of this paper does not permit a discussion of the whole subject, the following table is offered to show the more im-

portant conditions necessary for producing desired properties of concrete.

Property Desired	Conditions Necessary				
	3CS	Fineness	Mix	Cur. Temp.	Other Conditions
Early strength.....	High	High	Rich	High	
Final strength.....	—	High	Rich	Medium	High 3CS + 2CS
Low shrinkage.....	High	Medium	Lean	High	Aggregate important
Low temp. (Mass)...	Low	—	Lean	Low	Slow casting and low initial temp.
Low temp. (Thin)...	Low	Low	Lean	Low	Low initial temp.
Durability.....	Med.	High	Rich	—	Dense concrete

LOW 3CA AND LOW WATER CONTENT INDICATED FOR ALL DESIRED PROPERTIES.

The above table does not include all properties of concrete nor does it include all conditions. Furthermore, many qualifications could be stated. For example, a lean mix is indicated for low shrinkage, but if the mix is too lean, the shrinkage is often increased by the resultant increase in entrained air. The interesting feature of the above table is that it shows the futility of obtaining the maximum quality in all properties simultaneously. For example, high early strength demands high 3CS and a rich mix, while low temperature rise demands a low 3CS and a lean mix. There are shown only two conditions that lead to universally desirable properties; they are low 3CA content in the cement and low water content in the concrete. The problem of the engineer is to determine the best combination of conditions that will give best all-round properties for a particular structure. To the above list of properties, the engineer will add the cost, which is often most important of all.

PROBLEMS IN IMPORTANT STRUCTURES.

Consider next the design of cement and concrete for important structures, where only a slight improvement in quality has a great money value in increased safety and durability. The Boulder Dam is an outstanding example. Here is a mass of concrete 600 feet thick at the base and 725 feet high. If ordinary concrete were used and it were cast as in smaller dams, the temperature changes would cause serious

cracks and the dam would not safely resist the extraordinary water pressure. More important, the dam would cool at a gradually decreasing rate and only after about 100 years would the final temperature be closely approached. Filling of cracks and joints with cement grout before cooling is complete would be futile, and American patience and time valuation would not permit waiting many years before placing the dam in service.

The most obvious solution of the temperature problem in Boulder Dam could not be adopted because of cost and lack of precedent. The ideal plan would have been to cast a concrete of low heat generation at low enough temperature so that the heat generation would bring the concrete approximately to the yearly average air temperature. There would then be no tendency for cracks to develop, joints would be unnecessary and strength would only need be high enough to provide a reasonable factor of safety, that is, the strength would only need be three or four times the greatest stress the concrete would be expected to carry. This plan would have necessitated (1) a very-low-heat cement, (2) a lean, dry mix, (3) refrigeration of water and aggregates before mixing concrete, and (4) suspension of concreting during the hottest summer months. A cement content of not over 3 sacks per cubic yard of concrete would have been necessary to keep the temperature rise sufficiently low. While good concrete can be made with this cement content where good gravel of 6-inch maximum size is available, the use of such concrete is unprecedented for service comparable with that to be encountered by Boulder Dam.

A second plan was to employ precast blocks of fairly large size and place them after they were cooled. Estimates showed that the cost of storing and handling such blocks would be too high.

Finally, the plan of using a low-heat cement and removing the heat from the concrete by passing refrigerated water through embedded pipes was adopted. The dam was divided into many individual monoliths by contraction joints running both ways. Final temperature was obtained by the artificial cooling as soon as the concrete was well hydrated. This caused an opening of the joints, which were duly filled with

cement grout under pressure. Careful attention was given to every detail and the plan proved entirely satisfactory. The grouting problems were solved by laboratory and field researches and a continuous check was made of internal temperatures and joint openings through the use of electrical instruments.

Some may wonder why a low-heat cement was used in conjunction with the artificial cooling. The refrigeration could not be applied for a few days after the concrete had been cast and a large part of the temperature rise had then already occurred. If a high-heat, or normal, cement had been used, there would have been cracking troubles in each of the monoliths that made up the dam. With the low-heat cement these troubles were largely eliminated. The heat problem was especially severe because the average night and day temperature for an entire month was about 100 degrees in the middle of summer.

The purpose of the artificial cooling was not primarily to keep the temperature from rising but rather to remove the heat and get the dam to its equilibrium temperature at an early age.

Cement specifications were written only after an extensive research program involving tests on many thousands of specimens. About 90 cements of varying composition and fineness were tested under a variety of conditions simulating those expected to obtain in Boulder Dam. It was essential that the cement chosen would make a durable concrete, and that not too much heat would be liberated after the artificial cooling was stopped. In fact, a cement was sought that would have the lowest heat of hydration consistent with good strength, impermeability, and durability without exceeding the chosen cement content of one barrel per cubic yard. But safety was so important that no cement composition could be considered that had not at some time or other seen actual service in a structure.

For winter concreting, standard cement was found to be adequate but for concreting during all of the warmer months, special low-heat cement was necessary. Specifications for the low-heat cement differed from those of standard cement as follows:

Maximum limits were placed on the four major compounds as follows: 3CS—40 per cent., 2CS—65 per cent., 3CA—7 per cent., and 4CAF—20 per cent.

The loss-on-ignition limit was lowered from 4 per cent. to 3 per cent.

The heat of hydration was limited to 65 and 75 calories per gram at ages of 7 and 28 days, respectively (the heat of hydration of a standard cement when tested according to the method specified is about 90 calories per gram at 28 days).

The fineness of the cement was not permitted to be less than 2300 when tested by turbidimeter (in comparison, standard cement, for which there is no effective limit, averages only 1400 square centimeters per gram).

A low requirement for compressive strength of mortar cylinders was specified, 1,000 p.s.i. at 7 days and 2,000 p.s.i. at 28 days with the additional requirement that whatever the 7-day strength, the 28-day strength must be at least 35 per cent. higher.

The problems in the Grand Coulee Dam on the Columbia River in Washington are similar to those in Boulder Dam. The Grand Coulee Dam eventually will require four times as much concrete as the Boulder Dam but will not be as high. Again artificial refrigeration is applied by passing cold water through pipes cast into the concrete. Extremely low-heat cement is not specified in this case because of the colder climate. Therefore the specifications for the cement permit up to 55 per cent. of 3CS to keep it from being too slow hardening for the cold weather. The limit on 3CA content is again fixed at 7 per cent. The limit on ignition loss is still further reduced to avoid the possibility of getting prehydrated cement. No upper limit is placed on fineness, but the minimum requirement is raised to 1,800 sq. cm. per gm. The increased fineness is consistent with the knowledge that finer cements are generally more efficient and advantageous except for certain cement compositions and for structures of intermediate thickness where the early rate of liberation rather than the total amount governs the cracking. The compressive strength requirement on mortar cylinders was correspondingly raised for the Grand Coulee cement to 1,500 and

2,000 p.s.i. at 7 and 28 days, respectively. In case of an abnormally high 7-day strength, the 28-day strength was then required to be only 25 per cent. higher than the 7-day strength.

In dams of moderate size and in semi-massive structures of all kinds, studies are now being made to determine the most economical method of preventing cracks and large stresses due to temperature changes. The installation of refrigeration plants and the embedment of cooling pipes is expensive in any case, but is relatively more so in smaller structures. The solution seems to lie in (1) obtaining the concrete having the lowest heat of hydration consistent with good quality and (2) casting that concrete at a variable rate. A method has been developed for predicting the temperature changes in a structure of almost any size or shape if sufficient information is available. Knowing the temperature changes, approximate stresses can then be computed by taking into account the conditions of restraint and the changing plasticity of the concrete. It is not the intention here to explain these computations in detail but rather to sketch the method and tell what the computations show. It is shown that rather large temperature changes can be withstood in the central portion of a massive section of concrete if the boundary conditions are favorable. Thus in a dam, it is indicated to begin by casting several shallow lifts of concrete and allowing each lift to cool several days before adding the next. When about 20 feet of concrete have been cast, the lifts can be made thicker and the construction speeded up. A great advantage can be achieved without slowing the average rate of construction. Advantage is taken of the fact that some distance away from a foundation there is a sympathetic expansion and contraction of the various lifts of concrete, while at the foundation, the concrete tries to expand and contract without finding much sympathetic movement of the foundation. Furthermore, advantage is taken of the fact that several hundred pounds per square inch are required to crack concrete but if a crack is already started only a few pounds per square inch will cause it to continue. It is therefore important to prevent a crack from starting where tendency to crack is greatest, that is, at the boundary between concrete and rock. Thus, in the interior of a mass of concrete, a tensile stress of 300 p.s.i. may cause no damage and in fact the concrete will gradually

deform plastically to reduce this internal stress. The combined plan of controlling the casting rate and using a low heat cement and a lean concrete mix seems to be the economical solution of the heat problem in many massive structures. The research worker would like also to include the requirements of precooling the concrete materials before mixing, and suspension of concreting during the hot months, but he cannot definitely show a money advantage that is equal to the added cost.

In conclusion, a few words may be added regarding the current researches on the subject of drying shrinkage of thin structures. Progress is being made and it seems possible that sometime in the future concretes of low drying shrinkage will be obtained without great cost. This probably will not be accomplished by any material change in cement composition. Cements that give good strength do not differ greatly as to their shrinkage characteristics. The gel that provides the strength has thus far been made reasonably shrinkage free only by high-temperature curing or by prevention of drying. Pure cement pastes shrink several inches per hundred feet of length upon thorough drying at average humidity while concretes shrink about one-fifth as much. The aggregate in the concrete tends to reinforce the paste against shrinking and most aggregates do not do their job very well. Research workers are only beginning to learn how to design the concrete so that the aggregate can better reinforce the cement paste against shrinkage. Their findings have not yet been adopted. The low shrinkage obtained, for example, by using certain limestone aggregates shows what the aggregate can do when it is really effective. But the problem is far from solved yet and it is believed that cement research remains largely for the future.

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A QUANTITATIVE STUDY OF MOUNTAIN BUILDING ON AN UNSYMMETRICAL EARTH.

BY

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ABSTRACT.

The principle of isotatic equilibrium of the earth's crust is discussed and restated in a useful form which takes account of superposed vertical stresses.

A quantitative study of the deformations produced in the earth's elastic crust by tangential compressional forces is made. The calculated physical properties of a suboceanic compressional downfold agree well with those observed in certain long and narrow ocean deeps. The properties of a downfault in continental regions are similarly deduced and it is shown that such a deformation evolves ultimately into a great sedimentary prism or geosyncline. The processes which fold and ultimately uplift the prism to form a mountain chain are quantitatively considered. A quantitative theory of mountain building leads to a law of mountain heights in good agreement with observation. The considered mechanisms are consistent with the principle of isostasy and with the author's earlier estimates concerning the concentration of stresses at the boundaries of the great continents. It is concluded that the circum-Pacific mountain chains could only have been generated on an unsymmetrical earth produced probably by a fission process much like that suggested in the author's binary star theory of the origin of the solar system (*Phys. Rev.*, **39**, 130, 311 (1932)).

The structure and geological history of the great mountain ranges and ocean deeps have been the subject of considerable study. These studies have emphasized the complexity of their physical development, yet certain well established factors are characteristic of all systems and these provide a moderately reliable guide to a proper physical interpretation of their evolution. Contemporary theories of mountain building are most unsatisfactory, principally because they are not truly consistent with the requirements of isostasy, but also because of their qualitative nature. In order that the present investigation may be made quantitative, it is necessary to adopt somewhat idealized structures for calculation, but the essential physics is carefully preserved and the numerical estimates should apply, roughly at least, to actual mountain structures and deeps.

Foldings of the earth's crust take place on a grand scale only along geologically active belts which are more or less confined to long and moderately narrow regions that have a marked tendency to parallel the continental margins. The folds occur with seeming indifference on the continents or on the ocean bottoms but invariably the elevations and the accompanying depressions are nearly parallel and essentially linear in form. This latter characteristic is particularly evident in mountains of great height. Other geological facts are perhaps more remarkable. For example, in the larger mountain ranges, the deposited sediments are from 10 to 15 Km. thick, yet they were deposited in nearly every case in shallow seas or on alluvial plains. Observations show, too, that the evolution of mountain chains is initiated by a long period of slow depression and sedimentation and is ended abruptly by a geologically rapid vertical rise accompanied by tremendous folding and overthrusting.[1]

Gravity surveys yield other information of great value, for they show that the presence of great mountain ranges and the erosion, transport and deposit of great masses of sediments has little or no effect on the surface gravitational acceleration. This rather surprising fact is well established and is responsible for the development of the principle of isostasy.[2] This principle is founded physically on the presumed fact that the deeper and mechanically weak layers of the earth essentially preserve a hydrostatic equilibrium. Hence gravitational equipotential surfaces are also surfaces of equal pressure when mechanical equilibrium is finally reached, and therefore mass adjustments and transport above a selected equipotential will be such as to maintain an equal pressure over it. Thus, if no superposed vertical stress exists in the upper layers

$$p = Mg, \tag{1}$$

where p is the pressure over the selected equipotential, M is the mass per unit area above it and g is mean gravity within the layer. Certain practical considerations place a lower limit to the area which may be considered in applying the relation.[3] Now it is clear that the application of an external vertical stress to the crust will also contribute to the pressure at a selected equipotential surface so that in a crust which

may be subject to such vertical stresses the usual expression of Equation 1 is incomplete and the condition of isostatic equilibrium is specified more accurately by

$$S_v + Mg = \text{Const.}, \tag{2}$$

where S_v is the vertical stress. Again, for practical reasons, the equation must be applied to regions of reasonable extent. It must be understood that isostatic equilibrium is only approximately realized under actual circumstances due to the finite strength of the crust and the foregoing relations only specify the final equilibrium toward which the crust is constantly striving. It is a universal practice to express the pressure at the equipotential surface in terms of the mass above it so that if one follows the usual convention, S_v would be equivalent to a fictitious mass per unit area M' given by

$$M' = -S_v/g. \tag{3}$$

Therefore the gravitational anomaly Δg due to the fictitious mass and to any real excess mass is given approximately by

$$\Delta g = +2\pi\gamma\left[M_e - \frac{S_v}{g}\right], \tag{4}$$

where γ is the gravitational constant and M_e is the mass per unit area which is in excess of that producing isostatic equilibrium. In many cases it is impossible to separate the two components of the anomaly, but in undeformed regions of the crust there is reasonable assurance that S_v is not important. Therefore the scatter in the observed values of the anomaly in such undeformed regions set reasonable upper limits for the values of M_e. This scatter is equivalent to a layer of rock about 470 meters thick. But in special regions of the earth, anomalies 10 times as great are observed which probably should be attributed to superposed vertical stresses. Now Eq. 4 ignores the variations due to different distributions of density of the adjacent layers but does offer a convenient way of estimating the approximate vertical stresses which may be imposed on deformed regions of the crust. Moreover, since superposed downward stresses in special regions must usually be accompanied by upward stresses in not too remote adjoining regions, it is clear that the average gravitational

anomaly over a very large area must be quite small. Thus large negative gravitational anomalies due to vertically downward stresses are invariably accompanied by positive anomalies in adjacent regions. The important conclusion to be drawn from the gravitational and isostatic data is that under normal conditions an outflow of plastic rock in the deeper layers invariably accompanies the deposit of real or fictitious mass on the surface and the mass transported away is equal to the mass added above. Moreover, since the density of the rocks in the two regions is usually different, the level of the original surface will be altered by such a transfer. In the following paragraphs the term isostatic equilibrium will be used in the sense specified by the complete statement of Eq. 2.

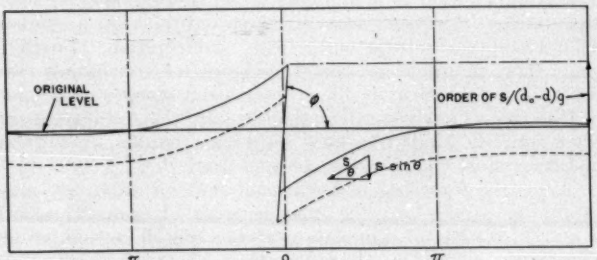
Certain great ocean deeps are remarkably long and narrow downward folds in the ocean floor which extend from 2 to 5 Km. below the surrounding level. The great deeps are invariably located in regions of orogenic activity and tend to lie parallel to the closely adjacent continental margin or to the direction of the folds in nearby mountains. A majority of geologists agree that the deeps are produced by a large scale downward faulting or folding and thickening of the underlying layers.[1] This conclusion is based principally upon the observed systematic negative gravitational anomalies which invariably are associated with very long and narrow deeps. The anomalies are usually somewhat positive in regions nearby the deep but are strongly negative immediately over or to the landward or steeper side of it. The non-coincidence of the geometrical and gravitational axes emphasize the fundamental nature of the downward compression and shows that the anomaly is not due simply to a slight superficial mass defect. Differences between the positive and negative anomalies as great as 400 milligals have been observed. Now geological investigations have usually interpreted the anomaly as a departure from isostatic equilibrium, but this conclusion is by no means necessary. Indeed, if most of the earth's crust is observed to be in isostatic equilibrium, at least to the approximation that local variations of gravity exceeding 20 milligals are not observed, no good reasons can be advanced to prove that highly specialized regions like the deeps are not

in equilibrium to at least the same approximation. Therefore, in this paper it will be assumed that the entire crust is essentially in isostatic and stable equilibrium at all times and therefore one may ignore the small contribution due to M_e in Eq. 4. Hence, employing the principle of isostatic equilibrium in its complete and correct form, the gravitational anomalies observed near certain deeps imply a downward stress on the outer crust of a magnitude given by Eq. 4. Thus, if we take $\Delta g = 0.300$ cm./sec.^{2}, $\gamma = 6.67 \cdot 10^{-8}$ and $g = 10^3$ cm./sec.^{2}, it is found that the typical downward stress is $7.5 \cdot 10^8$ dynes/cm.^{2} Such a stress is smaller but is of the order of magnitude of the tangential stress which is known to be necessary to produce folding in mountain chains. Therefore, in order to account for ocean deeps, one must show that tangential stresses may be converted to vertical stresses as a result of faulting and crustal deformation. Further, since there is ample geological evidence for concluding that orogenic development is due to tangential stresses developed in the crust as a whole rather than locally, the stability of the crust and its ability to pass on these stresses over great distances must receive some consideration.

With the foregoing observational data in mind, we may profitably proceed to the consideration of some idealized cases which probably simulate moderately well certain actual situations. Consider a great surface sheet of earth stuff subject to the vertical stresses of gravity and to superposed tangential stresses. If the sheet is perfectly uniform and accurately horizontal, it is in metastable equilibrium in relation to the tangential forces, but if small vertical deformations of the sheet are imposed due to any one of several different causes—tidal, sedimentation processes, non-uniform cooling or local failure of the sheet—then the original tangential stresses will evidently develop vertical components of determinable magnitude. For example, the tangential stress S throughout a deformed sheet is nearly enough its original value in undeformed regions and therefore the vertical component S_v is obviously given by $S_v = S \sin \theta$ where θ is the angle that the axis of the crust makes with the horizontal. Thus, in reasonably deformed regions, S_v is smaller but of the same order of magnitude as the tangential stress.

As another example, consider the geologically important case of Fig. 1 in which faulting of the layer of thickness T has taken place along the plane of maximum shear. One may properly neglect sliding friction in the following discussion because the cyclic deformation by the tides will cause a slow creep or slippage and the region will ultimately reach approximate equilibrium. Therefore, calling φ the angle that the faulted plane makes with the direction of the applied horizontal stress, the vertical force due to compression is $ST \cos \varphi \sin \varphi$ and since in the normal case of failure by shear φ approximates 45°, the vertical stress is something like one-half of the tangential stress. Further, if plastic flow

FIG. 1.



HOMOGENEOUS THRUST FAULT

is initiated it is clear that the stresses in all directions will be the same. In general, therefore, one may conclude that vertical stresses in the earth's crust may readily exist in special deformed or faulted regions which stresses are smaller but of the general order of magnitude of the tangential stress. One may therefore write

$$S_v = qS \qquad (5)$$

and note that q depends on the geometry of the particular distorted layers but in average cases perhaps lies between 1.0 and 0.2. The deformation of the crust outside of orogenic belts or thrust faulted regions is rather small and q there probably is small.

The next point of interest is the vertical stability of the compressed layer. The stratified nature of the outer layers of the crust and the fractures which invariably traverse it suggest that the layers simply lie one upon the other and are not physically bonded. That is to say, the surface layers are weak toward vertical tensional stresses. Thus if a vertical stress is applied at some point within a layer which exceeds the downward stress due to the weight of the material above it, then it is clear that the layer is unstable and movement will be produced which may ultimately result in folding. The critical case occurs when $S_v = tdg$ where t is the thickness of rocks above the "critical level." Whence by aid of Eq. 5 it is found that the "critical level" t is

$$t = \frac{qS}{dg}. \qquad (6)$$

One may therefore conclude that vertical instability in the outer stratified regions of the earth's crust due to tangential compression alone must be confined to surface layers above a "critical level" and that stresses below the critical level will be passed on undiminished to adjacent regions or may initiate downward plastic flow. If one adopts the case of plastic deformation as typical $q = 1.0$, $d = 3$ gms./cm.^{3}, and $S = 10^9$ dynes/cm.^{2}, then the critical level, according to Eq. 6, will be located at $t = 3.2$ Km., or if $S = 2.10^9$ dynes/cm.^{2}, then t will approximate 6.4 Km. Therefore, there is a possible element of justification for the attitude of certain geologists who have insisted that tangential forces adequate for mountain building could not be transmitted over great distance. We see, however, that their objection cannot be sustained in actual cases for no difficulty is imposed when the layers below 3 Km. are assumed to be the ones which actively pass on the tangential compressional stresses.

The foregoing mechanism of upward instability and folding as a result of compression may be reproduced in miniature by laying a previously crumpled sheet of writing paper flat on a table and applying a uniform compressional stress to its edges. In general, the writing paper will simply fold up in the middle producing a ridge at right angles to the applied stress and will not transfer an appreciable horizontal force.

However, if the writing paper is weighted down by a more or less uniform distribution of dimes or other light coins and a tangential stress again applied, it will be found that the stress which can be passed on is much greater than the initial value, but ultimately the definite irregularities in the paper will cause it to push up again, forming a ridge nearly at right angles to the applied force. When this ridge is pushed up, of course the stress is somewhat relieved and further compression simply distorts the initial ridge. Heavier coins may be placed on the paper and one may readily satisfy himself that the compressional stress passed on by the lower layers may be increased to the point of failure of the compressed sheet if an adequate stabilizing vertical load is placed on top of it.

FORMATION OF COMPRESSIONAL TROUGHS.

It is perhaps fair to say that very little of a definite nature is known about the strength of the earth's outer shells. It has been fashionable to assume that the earth's crust will deform plastically even under small or nominal stresses if these stresses are applied for a sufficient length of time. Such an assumption is probably quite wrong and while definite information regarding mechanical properties of the crust as a whole is lacking, little error will be introduced if it is assumed that the crust will *not* flow plastically until "one-sided" stress is applied to it which is of the order of the crushing stress of the material. In general, the critical stress at which "one-sided" failure takes place will change with temperature and with pressure, so that at depths of perhaps 500 Km. the earth's outer shell may flow plastically under a system of stresses which would not appreciably deform the outer layers. It is impossible to make definite statements in regard to the depth at which plastic flow under small stress will be initiated, in view of the contradictory geological and seismological data, for geology on one hand provides many examples of apparent plastic flow under stresses comparable to the crushing stress, while seismology shows that earthquakes may originate at depths of several hundred kilometers. To fix the ideas in this discussion, however, we choose to assume that the material of the outer layer of the crust is relatively strong and will support one-sided stresses of the order of 10^9 dynes/

cm.^{2} but will fail or flow plastically under greater stresses, while below perhaps, 100 Km., the strength decreases in an unknown and unspecified manner. It should be pointed out in this connection, however, that at least the outer ten kilometers is not a homogeneous shell but should be likened more to a mosaic structure in which many of the elements of the mosaic will support the stresses assumed, but marked bending stresses, shear or tension, applied to the whole may form fissures at the boundaries of the mosaic elements, and fracture will occur easily. Even if the crust were perfectly uniform and withstood stresses of 10^9 dynes/cm.^{2}, yet, it will fail by fracture if an overhanging or unsupported section exceeding about 3 Km. in width is developed. Therefore, large scale deformations of the crust are invariably accompanied by faulting and the entire region must always approach a vertical equilibrium.

The application of adequate tangential stresses to a uniform deformable sheet invariably leads to fracture and overthrusting or to the formation of one or more folds whose principal linear dimensions lie approximately at right angles to the direction of the applied stress. In the case of the earth's crust, tremendous tangential compressional stresses of the order of 10^9 dynes/cm.^{2} are set up and these being concentrated usually near the boundary of the continents produce flextural deformation and ultimately cause the failure of the crust at some especially weak region. Alternately the underlying layers may fail along shear planes and override, or may simply experience a fracture along the vertical with subsequent downward development. Any one of the suggested processes may produce a small down-warping of the crust and if the angle that the down-warped sheet makes with the horizontal is increased, the vertical component of the applied stress to the sheet increases. Thus, a small initial upward or downward deformation is initially unstable and proceeds to an equilibrium. The deformations upward bring about erosion whereas the downward deformations result in sedimentation accompanied by downward instability. Therefore downward folds accompanied by the deposition of sediments are of great geological interest. The conditions which must exist in idealized cases may be readily worked out. These ideal cases,

while never actually achieved, are nevertheless roughly reproduced and since they may be made quantitative, give a useful idea of the dimensions, depth, sedimentation, etc., which may be encountered in any actual geosynclinal development. As pointed out in earlier paragraphs, the outer one or two kilometers of the crust do not pass on tremendous tangential stresses, but since these layers lie more or less evenly upon the deeper stressed layers the figure of the surface and deeper layers will be substantially the same.

First consider a homogeneous and elastic strip of the earth's crust one centimeter wide which lies in the XY plane and is subject to a compressional stress S. Let the thickness of the strip be T, its density d_0, modulus of elasticity E, and I its section moment of inertia. Suppose that the strip is essentially a loaded beam with its neutral axis initially lying along the x axis and that a compressional force ST is applied to its ends. Evidently the beam will be deformed and develop transverse displacements along the y axis. These displacements are resisted by a hydrostatic counterstress d_0gy which represents the weight per unit area of the displaced subcrustal material or more generally by a counter stress $(d_0 - d)gy$ in case the deformation is filled completely with sediments of average density d. The moment of these forces about a point x, y is [3]

$$M_y = (d_0 - d)g \int_{q=0}^{q=x} y_q(x - q)dq, \tag{7}$$

where q is a second variable measured along the x axis. Moreover, the moment due to the deformation of the elastic beam is given by the classical expression

$$M_b = EI \frac{d^2y}{dx^2} = \frac{ET^3}{12} \frac{d^2y}{dx^2}. \tag{8}$$

If the beam is to be in equilibrium, the sum of the moments of all the acting forces about the point x, y must be zero, whence

$$(d_0 - d)g \int_{q=0}^{q=x} y(x - q)dq + \frac{ET^3}{12} \frac{d^2y}{dx^2} + STy - Fx = 0, \tag{9}$$

where F is the vertical shearing force at the origin which acts on the beam. Differentiating this twice

$$\frac{ET^3}{12}\frac{d^4y}{dx^4} + ST\frac{d^2y}{dx^2} + (d_0 - d)gy = 0. \qquad (10)$$

The solution of this equation is of the form $y = Ae^{mx}$ where

$$m = \pm\sqrt{\frac{6S}{ET^2}}\left[\pm\sqrt{1 - \frac{(d_0 - d)gET}{3S^2}} - 1\right]^{1/2}. \qquad (11)$$

Two types of solutions result according as the roots are real or imaginary. If

$$S < \sqrt{\frac{(d_0 - d)gET}{3}}, \qquad (12)$$

the solution is quasi periodic and represents an upward or downward fold in the crust which is believed to be of considerable geological importance. On the other hand if

$$S > \sqrt{\frac{(d_0 - d)gET}{3}}, \qquad (13)$$

the solution is truly periodic and represents a wave-like deformation of the crust. Note must be made of the fact that for upward deformations without erosion $d = 0$ while for downward deformations $d = 0$ only in case no sediments are deposited. Therefore it is clear from these relations that erosion and especially sedimentation processes accentuate any initial vertical instability. To evaluate Eq. 12, take $E = 5 \cdot 10^{11}$ dynes/cm.^{2}, $T = 10^7$ cm., $d_0 = 3$ gms./cm.^{3}, and $d = 2.7$ gms./cm.^{3}. It is found that the radical of Eq. 12 is $2.2 \cdot 10^{10}$ dynes/ cm.^{2}, therefore since independent evidence shows that the actual tangential stress in the crust approximates 10^9 dynes/cm. and this in turn approximates the crusting strength of most rocks, it is clear that the solution corresponding to Eq. 12 is the only one of physical importance unless: (*a*) the elastic modulus of rock is several orders of magnitude less than that assumed, or (*b*) the crust is made up of many layers of thickness less than a hundred feet.

A general solution of Eq. 10 corresponding to the physically

interesting condition expressed in Eq. 12 is

$$y = Ae^{ax} \sin (bx + c_1) + Be^{-ax} \sin (bx + c_2). \qquad (14)$$

In cases of geological interest the extent of the compressed layer is very large and therefore $A = 0$. Imposing the boundary conditions, at $x = \infty$ one takes

$$y = \frac{dy}{dx} = \frac{d^2y}{dx^2} = 0,$$

and making use of the fact that

$$M = EI\frac{d^2y}{dx^2} \quad \text{and} \quad F = \frac{dM}{dx}$$

we have, at $x = 0$,

$$\frac{d^3y}{dx^3} = \frac{12F}{ET^3} = aB(3b^2 - a^2),$$

thus it is found that

$$y = \frac{12Fe^{-ax} \cos bx}{ET^3a(3b^2 - a^2)}, \qquad (15)$$

where F is an externally applied vertical shearing force at the origin and

$$a = \pm\left[-\frac{3S}{ET^2} + \sqrt{\frac{3(d_0 - d)g}{ET^3}} \right]^{1/2}, \qquad (16)$$

$$b = \pm\left[\frac{3S}{ET^2} + \sqrt{\frac{3(d_0 - d)g}{ET^3}} \right]^{1/2}. \qquad (17)$$

Now the solution of Eq. 14 is physically significant only when a fault occurs at the origin in such a manner that a vertical force is applied to the end of the beam. One case of geological interest corresponds to the type of thrust fault shown in Fig. 1 and here obviously the vertical shearing force is given by $ST \cos \varphi \sin \varphi$. Now the substitution of numerical values in Eqs. 16 and 17 show that the first terms are but a few per cent. of the remaining ones, therefore to a fair approximation

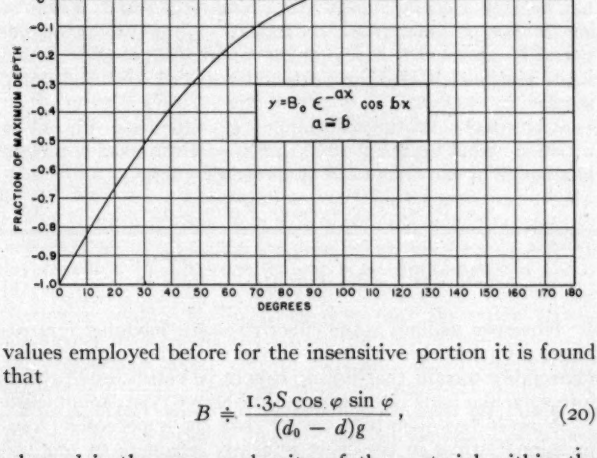
$$a \doteqdot b \doteqdot \left[\frac{3(d_0 - d)g}{ET^3} \right]^{1/4}, \qquad (18)$$

whence Eq. 15 becomes finally

$$y = 2\left[\frac{3T(d_0 - d)g}{E}\right]^{1/4} \frac{S\cos\varphi\sin\varphi}{(d_0 - d)g}\,\epsilon^{-az}\cos bx. \quad (19)$$

The form of this equation is plotted in Fig. 2. It is convenient to note that the maximum depth of the compressional trough is the product of an insensitive factor and one that is quite sensitive to the physical quantities. Substituting numerical

FIG. 2.



values employed before for the insensitive portion it is found that

$$B \doteqdot \frac{1.3S\cos\varphi\sin\varphi}{(d_0 - d)g}, \quad (20)$$

where d is the average density of the material within the trough. Adopting as before $S = 10^9$ dynes/cm.^{2} $(d_0 - d) = 3$ gms./cm.^{3} (i.e. assume no sedimentation), $E = 5\cdot 10^{11}$ dynes/cm.^{2} and $T = 10^7$ cms., it is found that a compressional downward deformation is produced on one side of the origin and a mirror image upward deformation on the other side whose maximum deformation in depth is 2.17 Km. and width is approximately $\pi/2b$ or 242 Km. The upward fold is of

the same magnitude and hence the maximum difference in height between the upraised fault scarp and the depressed fold approximates 4.2 Km. If the downward fold becomes filled with sediments it is clear that it will develop downward and simultaneously widen. Numerical estimates of the final dimensions may be found in a later section. It is interesting to note further that the gravity anomaly in this type of fold before erosion or sedimentation take place approximates $\pm$ 200 milligals at the thrust fault, decreases to zero and changes sign at about 250 Km., increases again to a maximum of $\pm$ 12 milligals at about 500 Km. and decreases finally to zero at about 750 Km. The curve of gravity anomaly is similar in form to the actual figure of the deformed crust which has been plotted in Fig. 2. In a following section it will be shown how sedimentation processes can bring about an important downward development of the depressed section while erosion but slightly reduces the altitude of the elevated section.

Consider next the solution of Eq. 10 when Eq. 13 is satisfied rather than Eq. 12. In this case the crust is thrown in a series of wavelike folds of the general form

$$y = B_3 \sin (b_2 x + c_1) + B_4 \sin (b_3 x + c_2), \qquad (21)$$

where

$$b_{2,3} = \left[\frac{6S}{ET^2} \left(1 \pm \sqrt{1 - \frac{dgET}{3S^2}} \right) \right]^{1/2}. \qquad (21a)$$

However, as long as the effective elastic modulus approximates that assumed, namely $E = 5 \cdot 10^{11}$ dynes/cm.^{2} it seems reasonably certain that Eq. 13 cannot be satisfied and therefore wavelike folds in the crust of the above type cannot exist.

Now it has just been noted that the superposed compressional stress in the crust is nearly adequate to produce failure or plastic flow in the deeper layers and of course the outermost layers in many cases are traversed by many faults so that in either case the elastic stiffness of the crust may become small. The effective elastic stiffness of the crust evidently will decrease regularly as more and more faults are developed in it and in the limiting case we may obtain the solution of Eq. 10 by ignoring the first term. The solution corresponds to a wavelike deformation of wave-length com-

parable to the thickness of the layer of the form

$$y = B_5 \cos (b_5 x + c), \tag{22}$$

where

$$b_5 = \sqrt{\frac{dg}{ST}}, \tag{23}$$

so that the distance between crests λ of the deformation is

$$\lambda = 2\pi \sqrt{\frac{ST}{dg}}. \tag{24}$$

The applied and reacting forces in this type of fold are such that it is essentially unstable and when accompanied by adequate horizontal movement finally reaches equilibrium only when the originally horizontal layers are up-ended and the strata approach the vertical. The numerical estimates show that the above considered type of folding will take place in the crust if plastic flow or crustal failure occur on a large scale. Probably this type of folding takes place in most regions of very great orogenic activity and evidently is of the type which would permit the injection of considerable igneous rock. Now when the rock material is stressed beyond the elastic limit it is clear that the waves no longer necessarily extend deep into the crust but may be confined to the surface layers which we have seen can seldom support such a stress and therefore normally will not flow plastically. Indeed, the discussion leading to Eq. 6 suggests that the outer 2 or 4 kilometers may be sheared off by the plastic deformation of the supporting layers and thrown into sharp and greatly compressed folds. Suppose that the material above the "critical level" specified by Eq. 6 is sheared off; then by Eqs. 6 and 24 a series of *upward* folds will be produced whose crests on the average are separated by a distance something like $2\pi/\sqrt{q}$ times the initial thickness of the layers above the critical level as given by Eq. 6. Substitution of numerical values leads to the conclusion that the crests of such folds would be separated by distances of from 12 to 50 kilometers. It is believed that folding of this special type may account for the production of some of the low mountain chains or perhaps more precisely "pressure ridges." Moreover, the mechanism

is probably of importance in the final compressional phase of the evolution of a great geosyncline into a mountain range.

One may conclude from this idealized treatment that superposed tangential stresses approximating 10^9 dynes/cm. in a homogeneous earth's crust are adequate to produce downward compressional troughs a very few kilometers deep and some hundreds of kilometers wide, but are definitely inadequate to throw the crust into a series of wavelike folds unless failure on a large scale or plastic flow is rather generally produced. In the event that plastic flow does take place, surface "pressure ridges" spaced on the average about 25 Km. apart may be expected. It is considered significant that the depths of the great downward folds are proportional to the compressional stress. We shall see presently that this fact has an important bearing on the production of geosynclines and of those mountain ranges which have a marked tendency to parallel continental boundaries.

The foregoing results may be readily applied to a calculation of the properties of a fractured downward compressional fold on the ocean bottom. The arcuate form of many mountain chains and the wavelike folding which is always present indicates that tangential stresses must have been produced in the crust which were adequate to cause its failure. Therefore one may adopt for the tangential stress in the crust adjacent the fold a value approximating the crushing strength of basalt or 10^9 dynes/cm.^{2}. Employing Eq. 19, taking $d_0 = 3$ gm./cm.^{3} and $d = 1$ gm./cm.^{3} because the fold is filled with water, it is found that the maximum depth below the surrounding regions in a "fracture trough" approximates 2.9 Km. while the width of the fold is roughly 90 times the depth or 270 Km. The length of the fold evidently depends only on the extent of the compressed region and therefore will be comparable to a continental dimension. Further, since the downward fold is produced by tangential and downward stresses, a deficiency of mass over the fold is to be expected. The expected gravity anomaly is specified by Eqs. 4 and 5, and upon the substitution of the foregoing values turns out to be -0.200 cm./sec.^{2} which closely approximates typical observed values. Moreover, due to the vertical stress on the overthrusting section a positive anomaly of

approximately the same magnitude may be expected adjacent to the deep. The present calculation predicts that the maximum negative anomaly will be greatest immediately over the region of maximum downward stress and this would presumably nearly coincide with the deep but the calculated fold has been idealized and an unsymmetrical compression or some sedimentation could very readily shift the position of the maximum anomaly toward the edge of the deep and reduce its magnitude. Comparison of the calculated dimensions and anomalies with observation shows that there is a remarkable resemblance in the calculated properties of a downward fold produced by compression and those of actual deeps. However, this is not conclusive and it is profitable to examine downward compressional folds developed on a continent.

FORMATION OF GEOSYNCLINES.

The formation of a downward thrust fault trough may as readily take place on a continent as on the sea bottom and the bottom of the trough in this case may very well be forced below sea level. For example, if one assumes a superposed tangential compressional stress of 10^9 dynes/cm.^{2} and takes $d = 0$ because the fold at first is filled with air, then by Eq. 19, the maximum depth of the selected typical fold turns out to be 2.1 Km. Such a compression is evidently sufficient to force most continental plateaus below sea level in long narrow troughs. Now if the trough is completely filled with water it provides additional loading and the trough may sink to a further maximum depth of 2.9 Km. In most cases the depression will be only partially filled so that the maximum depth will lie somewhere between the limiting values just calculated. It is important to note that the depths of the compressional troughs are proportional to the applied tangential stresses and these probably were much less in past ages than they are today.[6] Indeed if the earth was once liquid, it is known that such stresses would be entirely absent. Thus, one may properly suppose that the tangential stress and the maximum depth of a typical compressional fold will increase in an unspecified manner with increasing time.

Geological observation shows that the sediments in a geosyncline were invariably laid down in seas less than a

third of a kilometer deep or on alluvial plains and that the sediments often are from 10 to 15 kilometers thick at one edge, becoming gradually thinner toward the other edge. Probably one need not be disturbed by the usually emphasized fact that the sediments are deposited in shallow seas because sediments which are washed down from adjacent highlands will necessarily be deposited along the shores and shallow parts of the sea and thereby will build both upward and outward submerged alluvial plains even if the original sea was some two kilometers deep. The difficult matter to understand is the mechanism which permits the deposition of sedimentary layers of such great thicknesses when according to the requirements of isostatic equilibrium only two kilometers can be deposited before the level of the bottom is raised to sea level. For instance, suppose the density d of the sediments is 2.7 gm./cm.^{3}, while the density of the underlying rock d_0 is 3 gm./cm.^{3}; then if the sea is originally $z = 0.3$ Km. deep and filled with water of density d_w, a thickness of sediments equal to a maximum of $T = z(d_0 - d_w)/d_0 - d$ or only 2 Km. can be deposited. Thus an apparent contradiction between the requirements of isostatic equilibrium and observation is encountered. Jeffreys [4] has attempted to patch up the inconsistency by assuming that plastic outward flow takes place in some of the deep but intermediate layers. Such an assumption is physically inadmissible unless auxiliary and improbable hypotheses are introduced to provide special regions toward which the plastic rock may flow.

The apparent contradiction of the foregoing paragraph is immediately resolved if one considers that the compressional and tangential forces in the crust produce downward vertical stresses and folds. As a specific illustration, consider the "fracture trough" having the figure of Eq. 19 and assume for some period of geosynclinal development that the horizontal stress in special regions of the earth's crust increased more or less regularly from, say, $3 \cdot 10^8$ to 10^9 dynes/cm.^{2}. In the subsequent section on the compressional stresses the justification for this assumption will be outlined. It is not a necessary assumption but is introduced because it rationalizes the interpretation. During the early state of compression, a downward trough due to underthrusting of depth given by

Eq. 19 and an upward fault scarp is generated. If we take $S = 3 \cdot 10^8$ dynes/cm.^{2}, and $d_0 - d = 3.0$ grams/cm.^{3}, the depth of the trough is found to be 0.63 Km. Suppose that this down-faulting submerges the very low continental plateau and it fills almost completely with water. With the same applied stress, but now with $d_0 - d = 2$ gm./cm.^{3}, the new depth below the surrounding land is something less than 0.88 Km. At the same time that this is taking place the reacting force on the overthrusting section produces an exactly equal vertical stress which raises this section 0.63 Km. above its original level, producing thereby closely adjacent highlands. Erosion and sedimentation set in and the material in the elevated highland is transported to the lower fracture trough; rapidly at first due to the large slopes but more slowly systematically as the elevated section is eroded away. In this manner successive beds of gravel-like conglomerates, then shales and limestone would be laid down producing systematic distributions in the sedimented trough. Now as sediments are washed into the trough it sinks forcing out the underlying plastic rock which is transported to the adjacent region underlying the rising segment and there assists in the rapid establishment of isostatic equilibrium. If the trough is completely filled and the difference in density between the sediments and the underlying rock is 0.3 gm./cm.^{3} then the final equilibrium depth is evidently 3.6 Km. according to Eq. 19. It is clear that throughout this interval the sediments are deposited in shallow water of depth varying regularly from a maximum of 0.88 Km. down to zero. Reference to Fig. 2 shows that most of the deposit will be made in waters not exceeding 0.5 Km. Meantime, the outer shell of the earth will have cooled still more and the compressional stress will have increased. Therefore, the equilibrium depth is greater than that just calculated so that the deposited sediments will still be precipitated in shallow water. Thus, as sediments are steadily deposited the tangential stresses constantly provide a downward deformation which on the average may readily maintain the sea bottom at nominal depths below the surface. Nor does the adjustment between rate of deposit and the necessary tangential stress appear at all critical because alluvial plains may be formed at the edge of the inland sea

and the width of the sedimented prism may be greatly increased even with a constant compression. At the same time that the downward fold is being filled with sediments, the raised and eroded adjacent section will continue to rise and presently all the surface layers will be carried away exposing ultimately the underlying igneous rocks.

It is the province of the geologist to trace the evolution of the sedimented prism in great detail, but one may see physically that the compressional stress may not be uniform over great geological epochs and as failure perhaps occurs in the crust of neighboring regions or secondary failure and sliding occur along the original fault, the entire sedimentary prism may rise a distance comparable to $S/d_0 g$ only to be forced downward upon the reapplication of the tangential stress. Thus, the exposure and flooding of an active region may take place many times in the course of the development of the sedimentary prism. Moreover, since adjacent regions are raised simultaneously, a cyclical deposit of characteristic sediments may be expected. The dimensions of the prism when the sediments are finally built up to adjacent levels may be estimated. In this final state it is probably correct to take the tangential stress as $S = 10^9$ dynes/cm.^{2} because it is known that folding of the sedimentary prism is extremely common. Adopting also $d_0 - d = 0.3$ gm./cm.^{3}, it is found by Eq. 19 that the maximum depth of the deposited sediments is 12.0 Km. and the width of the prism is 440 Km.

Now as the compressional trough develops downward as a result of sedimentation it is clear that the crust is further and further deformed and will finally fracture where the curvature is a maximum. It is easily shown that the greatest curvature for the crustal deformation specified by Eq. 19 occurs near $x = \pi/2b$ or at the edge of the trough. Now the stress at the extreme fibre of the layer is given by the well-known relation

$$S_x = \frac{ET}{2}\frac{d^2y}{dx^2} = BETab\epsilon^{-\frac{\pi a}{2b}} \qquad (25)$$

and therefore, by Eqs. 19 and 20, increases linearly with the depth of the compressed trough. Substitution of numerical values shows that the stress at the extreme fibre S_x approximates 10^9 dynes/cm.^{2} when B has increased to 7 kilometers.

Thus the downward development of the fold by sedimentation in most cases results ultimately in a second failure of the crust. The direction of the thrust fault in this second failure determines the final development of the sedimentary prism. If the new thrust fault forces the sedimented section downward, still more sediments may be accumulated. For example, if two thrust faults are produced in the crust and each tends to force the intermediate section of the crust downward it is easily shown that the intermediate section is forced down an average distance B_d before equilibrium is restored, where

$$B_d = \frac{ST(\cos\varphi \sin\varphi + \cos\varphi_x \sin\varphi_x)}{(d_0 - d)gp}. \qquad (26a)$$

Here p is the width of the depressed section of the crust and φ_x is the fault angle at the new break. Substitution of numerical values in this relation, taking $\varphi = \varphi_x = 45°$ and $p = 330$ Km., it is found that the average depth of sediments for equilibrium is 10.0 Km. while under different circumstances the maximum depth may approach twice this. If, on the contrary, the new thrust fault tends to lift the fractured segment, the lower end is further depressed and, upon the deposit of sediments, sinks to about 20 Km. while the newly faulted region is upraised about 5 Km. forming a temporary mountain. It can be shown in this case that the maximum thickness of the sedimentary prism B_u is given very approximately by

$$B_u = \frac{3ST\cos\varphi \sin\varphi}{(d_0 - d)gp}\left[1 + \frac{\sin 2\varphi}{\sin 2\varphi_x}\sqrt{\frac{(d_0 - d)}{d_0}}\right]. \qquad (26b)$$

Attention is directed to the fact that the mean depth of sedimentation in every considered case (see Eqs. 20, 26a and 26b) is a large fraction of $S/(d_0 - d)g$ or 30 Km. so that the actual geological structures probably are of this same approximate depth.

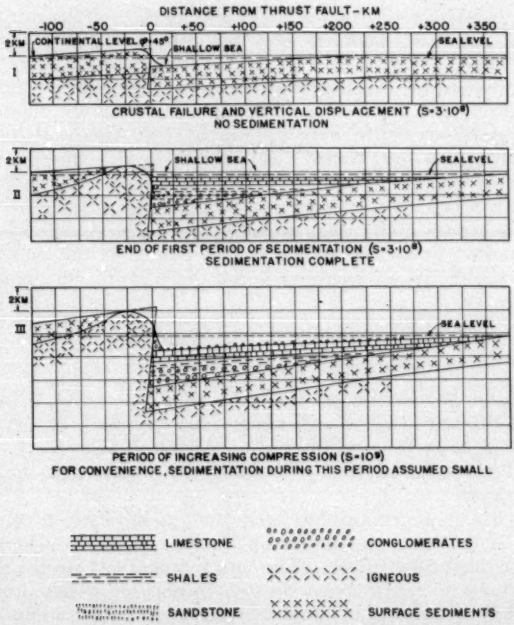
In Fig. 3 an attempt has been made to indicate, to approximate scale, the more important epochs in the development of a sedimentary prism. The exact numerical results probably have no significance since S is not accurately known, but they do show, I think, beyond reasonable doubt that tangential stresses in the crust make possible the generation

of great sedimented prisms much like those actually observed. Moreover, the deposition takes place in a manner which is entirely consistent with the known principle of isostatic equilibrium. It is thus clear that a constantly compressed downward fold has just the properties that are necessary to explain the type and character of sediments deposited in a typical geosyncline.

FORMATION OF MOUNTAINS.

The thick geosynclinal prism of sediments deposited in the highly specialized manner outlined above develops into a

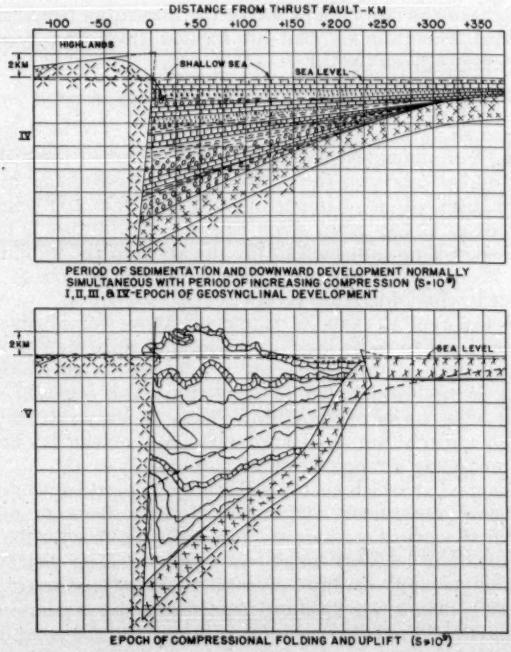
FIG. 3.



mountain range by a dual process long recognized by geologists. The sedimentary prism may be thrown into folds by tremendous tangential stresses and subsequently or possibly simultaneously raised by the vertical forces establishing isostatic equilibrium.

Consider first an unfolded prism of sediments. As the sedimentation of the compressional trough proceeds the deformation of the original layers becomes greater and greater, perhaps forcing back the shallow margin of the depression and ultimately fracturing in such a manner as to disrupt the original pattern of downward compressional stress.

FIG. 3.—*Continued.*


DISTANCE FROM THRUST FAULT-KM
-100 -50 0 +50 +100 +150 +200 +250 +300 +350
HIGHLANDS
SHALLOW SEA
SEA LEVEL
2 KM
IV
PERIOD OF SEDIMENTATION AND DOWNWARD DEVELOPMENT NORMALLY SIMULTANEOUS WITH PERIOD OF INCREASING COMPRESSION (S×10³)
I, II, III, & IV-EPOCH OF GEOSYNCLINAL DEVELOPMENT
2 KM
SEA LEVEL
V
EPOCH OF COMPRESSIONAL FOLDING AND UPLIFT (S×10³)

As a result of this local failure or the release of the compression by a failure in some other place, the sedimented trough of thickness H is no longer maintained in stable equilibrium by the downward applied stresses and it will rise as the dense underlying rock flows in to reëstablish isostatic equilibrium. Thus the work necessary to raise the sedimented prism above its surroundings comes from the work done in forming the original down fault. Its magnitude is given by Eq. 36. Upon the release of the superposed vertical stress, the weights per column of unit cross sectional area in various regions are approximately equal. Now, since the weight per unit area in the sedimented region is Hdg, while that nearby is $h_1 d_0 g$ and equal to it, we may eliminate h_1 and find that the difference in height of the two regions Δh_1 or the maximum height of the unfolded mountainous plateau formed from the synclinal sediments is given by

$$\Delta h_1 = H \frac{d_0 - d}{d_0} = \frac{fS}{gd_0}, \tag{27}$$

where f is a numerical constant less than but of the order of unity and depends on the geometry of the particular crustal deformation. It is of importance to note that the height of the plateau at any point when isostatic equilibrium is finally established is proportional to the thickness of the sedimented layer H at that point and to its deficiency of density with respect to the underlying layers.

In the typical example employed above for illustration, H turned out to be at least 12.0 Km. and since $d_0 - d = 0.3$ gm./cm.^{3} and d_0 approximates 3 gms./cm.^{3} the maximum height of the sedimented plateau above the original continental level turns out, by Eq. 27, to be at least 1.2 Km. Erosion will cut up such a plateau and reduce its height, thus producing a range of low mountains.

The rate at which such a mountainous plateau would rise is of some importance. Of course, if the tangential and the associated vertical stresses are released very slowly, the rate of rise might be very small, so small, in fact, that great rivers might very readily lower their channels more rapidly than the plateau was uplifted, thus giving the appearance of their having flowed over mountain ridges. On the other hand,

if the compressional stress is released all at once, the rise will take place in two steps. First by a very rapid rise due to the release of the vertical elastic forces, and second by a moderately slow adjustment to isostatic equilibrium of considerable magnitude which it is estimated will take about $2 \cdot 10^4$ years.

Consider next the increase in altitude due to the process of folding alone. Suppose that the horizontal stress is steadily increased to the point of failure or plastic flow of the rock. The discussion of Eq. 22 shows that the sediments will be thrown into a wave-like configuration and the sediments folded. Consider an arbitrary width of sediments w_1 as laid down originally and assume that compression and folding forces these sediments to occupy a narrower space w_2. Then we may write

$$(h_2 + H)w_2 \doteq \beta H w_1, \tag{28}$$

where β represents the ratio of the average to maximum initial thickness of the sediments.

Now the additional load per unit area caused by folding is evidently $h_2 d$ and according to the principle of isostasy this forces out plastic rock below of equal mass or $h_3 d_0$. Let the difference in elevation due to this process alone be Δh_2; then eliminating h_3 and noting Eq. 28, we have

$$\Delta h_2 = h_2 \frac{d_0 - d}{d_0} = H \frac{\beta w_1 - w_2}{w_2} \cdot \frac{d_0 - d}{d_0}. \tag{29}$$

Of course, in the absence of isostatic adjustment the height of the folded mountain would be simply h_2.

The total height H_T of the mountain above the original continental level is evidently the sum of Eqs. 27 and 29, or

$$H_T = H \frac{\beta w_1}{w_2} \cdot \frac{d_0 - d}{d_0} = Q \frac{d_0 - d}{d_0} = \frac{\beta f S w_1}{d_0 g w_2}, \tag{30}$$

where $Q = \beta H w_1 / w_2$ and represents the actual thickness of the folded sediments.

This relation contains an approximation to Washington's law [5] as a special case, since it predicts that the heights of the mountains are inversely proportional to the density of the underlying rocks. The relation is more general and may

well be expressed in the form of a law of mountain heights as follows: The heights of typical mountains which are in isostatic equilibrium are proportional to the depth of sedimentation in the original geosyncline, to the difference in density of the mountain sediments and the rocks below, to the ratio of the initial and final linear dimensions of the folded layers in the direction of the applied compressional stress and inversely proportional to the density of the rocks underlying the mountain. Alternately it may be expressed—The maximum heights of typical mountains are proportional to the tangential compressional stresses effective in the production of the parent geosycline, to the ratio of the initial and final linear dimensions of the folded layers and inversely proportional to the density of the underlying rocks. The first statement of the law may be established on purely observational grounds and indeed is principally an expression of the principle of isostatic equilibrium. The second form implies a mechanism of geosynclinal formation like that just considered.

It is interesting to note that the relations require no folding of the layers whatever to produce small mountains perhaps 2 Km. high but do require intense folding if high mountains several kilometers high are to be formed. This is in close agreement with observation. For instance, the Himalayan Mountains rise about 5 Km. above the adjacent continental level. Therefore, adopting the same numerical data used previously, it is found that in this mountain range w_1/w_2 must approach 4.0, or the compressional shortening of the crust is 75.3 per cent. Such a value may be reasonable. It should be emphasized here, however, that tangential stresses approximating 2 or $3 \cdot 10^9$ dynes/cm.^{2} could have generated the Himalaya's without the assistance of great compressional shortening.

The generation of a mountain chain by compression alone is a physical possibility even though contemporary geological interpretation does not suggest that it is common. The continental surfaces are invariably covered by relatively thin layers of sediments and if these could be pushed together by adequate forces they would produce structures much like a folded mountain chain. Assume a discontinuity of strength in the outer crust, such, for example, as might exist at an

ocean boundary, and suppose that sufficient tangential stress is applied to shorten the underlying layers either by fracture or by plastic flow (i.e., assume the existence of one-sided stresses exceeding $S = 10^9$ dynes/cm.^{2}). Early paragraphs indicated that the outer two or three kilometers of the weaker section of the crust above the "critical level" would be unstable and therefore are likely to be thrown into vertical folds. Because of the shortening and downward plastic flow in the levels below the critical level, the weaker surface sediments will pile up against the stronger section of the surface crust of sediments, sink to the level demanded by isostatic equilibrium and thus produce a great sedimented prism, or a compression type mountain. Such a mountain will be characterized by extreme and complicated folding. Let t be the initial thickness of the surface sediments and, as before, w_1 and w_2 their initial and final dimensions in the direction of the applied stresses, then $w_1 t = w_2(t + h_2)$, where $(t + h_2)$ is the thickness of the piled-up sediments. Solving for Δh_5 the mean height of this type of mountain, as before, we have

$$\Delta h_5 = t \cdot \frac{w_1 - w_2}{w_2} \cdot \frac{d_0 - d}{d_0}. \tag{31}$$

There is an upper limit to the value of Δh_5 because vertical stresses are converted to tangential stresses in exactly the same manner as the reverse process discussed in an early paragraph. Thus when the superposed vertical stress due to the weight of the mountain is some appropriate factor times the applied tangential stress, tangential equilibrium in the outer layers is established and further folding stops. We may therefore write

$$h_5 \leq \frac{q'S}{dg}, \tag{32}$$

where q' is a geometrical factor less than but of the order of unity and of the same type as employed in Eq. 5. In view of the known thickness of continental sediments, a comparison of Eqs. 31 and 32 shows that compression ratios (w_1/w_2) in this type of mountain must approximate 8 or 10 if the compression is to produce the highest mountain specified by Eq. 32. It appears somewhat doubtful that compression of

this order has taken place in the major mountain chains. Therefore the purely compressional type of mountain is likely to be rather low during the present geological epoch unless we have underestimated the applied stresses and compression. However, if the tangential stresses in the earth's crust continue to increase with time as the considerations in the next section suggest, future geological development may very well lead to high mountains of the purely compressional type. This remark perhaps should be supplemented by the observation that typical geosynclines do not now appear to be common on the earth at this time, probably because the outer crust is becoming constantly stronger. While the purely compressional type of mountain may be only of theoretical interest, it deserves to be included in any discussion of mountain building because it provides an adequate and reasonable mechanism for concentrating great prisms of sediments in special regions and undoubtedly plays at least a modest part in the generation of these structures.

A problem closely allied to that of mountain building is the effect of erosion on the height of mountains. Erosion reduces the mass loading of the mountain and this must be replaced by plastic rock of greater density flowing in below the mountain. Under conditions of exact isostatic equilibrium the change in elevation is

$$\Delta h_4 = h_4 \frac{d_0 - d}{d_0}, \tag{33}$$

where h_4 is the thickness of the eroded sediments and Δh_4 is the mean reduction in altitude. Therefore, roughly, 10 Km. of sediments must be carried away before the mountain will be reduced 1 Km. in height. So long as isostatic equilibrium is preserved it is improper to attribute the continued rise of a mountain to erosion. Jeffreys has made[4]* such an interpretation in the case of the Arun River which cuts the Himalayan range apparently originally at great heights. A more rational explanation is that the Himalayan Mountains are still rising as a result of further folding or compression or because

* It is perhaps worth noting that the statements on this subject in Jeffreys' book are contradictory. On page 160 he says that erosion will lower the height of a mountain while on page 148 he states that erosion will raise them.

vertical superposed stresses are being relaxed. Either assumption is quantitatively satisfactory from a physical point of view and the final decision in this matter must therefore be made by the geologist.

ENERGETICS OF MOUNTAIN BUILDING.

It is evident that mechanical work must be done on the deposited sediments of a geosyncline to raise them to their final elevated position in a mountain. Any acceptable theory of mountain building must provide a quantitatively satisfactory description of the origin and transfer of the required mechanical energy.

It should be clear from the preceding paragraphs that the source of energy according to the present account is essentially gravitational in origin and is not related to local thermal or radioactive phenomena. More specifically the necessary energy is derived from the applied tangential forces which act through distances of the order of 10 Km. while buckling the crust downward. Because considerable mechanical work must be expended in the form of friction during compression and this cannot be quantitatively estimated, a very approximate calculation of the energy will suffice.

Consider a mountainous prism of sediments, triangular in form and of altitude h. Again consider a slab of this prism of unit width. If the density of the mountain is d and the width of its base is w, the work necessary to elevate this prism above the surrounding level is evidently its weight multiplied by the distance of its center of gravity above the surrounding country or

$$W_m = \frac{dg}{6} wh^2. \tag{34}$$

Now if the mountain is without folded layers it is clear that this energy must have come from the tangential compressional force in the original crust acting through a distance equal to the amount of crustal shortening. The crustal shortening L is evidently $B/\tan \varphi$ or by aid of Eq. 20

$$L = \frac{1.3S \cos^2 \varphi}{(d_0 - d)g}. \tag{35}$$

Therefore, since the applied force is ST the work done in compressing the down fault is

$$W_c = \frac{1.3S^2T\cos^2\varphi}{(d_0 - d)g}, \tag{36}$$

where T is the thickness of the layer. Because of losses in energy due to friction, the energy supplied must exceed that actually stored in the mountain by an unknown amount. To take account of this let

$$W_c = KW_m, \tag{37}$$

where K is greater than unity. Therefore, one finds

$$h\sqrt{w} = \frac{S\cos\varphi}{g}\left[\frac{7.8T}{Kd(d_0 - d)}\right]^{1/2}, \tag{38}$$

whence, adopting $S = 10^9$ dynes/cm.^{2}, $K = 3.00$, $\varphi = 45°$, $d = 3$ gms./cm.^{3}, $(d_0 - d) = 0.3$ gm./cm.^{3} and $T = 10^7$ cm., it is found that $h\sqrt{w} = 3.8 \cdot 10^9$ and therefore the work done by the compression is more than adequate to raise a triangular mountain 5.0 Km. high and 500 Km. wide at its base. These numbers probably represent minimum values because the assumption that $K = 3.00$ is equivalent to assuming that 66 per cent. of the supplied energy is dissipated in the form of friction.

Now if shortening of the sedimentary layer by folding is further accomplished, additional energy is provided by the moving crust. In this case the added energy is

$$W_f = ST(w_1 - w_2), \tag{39}$$

where w_1 and w_2 have the significance expressed in Eq. 28. Thus the total available energy is the sum of Eqs. 38 and 39. One may readily verify that in most cases the magnitude of Eq. 39 is comparable to that of Eq. 36 and therefore it may contribute in an important manner to the possible maximum height of a mountain chain.

In general, therefore, relations expressing the conservation of energy lead to mountain structures entirely consistent with those structures deduced on dynamical grounds.

THE COMPRESSIONAL FORCES.

The foregoing paragraphs consider a stressed earth crust which is in a normal state of non-uniform compression. This compression is produced in at least three different ways: (1) by the uniform shrinkage of the earth as a result of cooling; (2) by the reduction of area of the spheroid as the earth's rotation is reduced by tides; (3) by gravitational forces arising from the present non-uniform distribution of density in the outer shell of appreciable strength.[6]

The uniform shrinkage of the earth by cooling must have had some influence on the formation of earth features during the early stages of its development and may very well have produced low compressional mountains more or less at random over the face of the earth rather than in a systematic manner surrounding the Pacific. Moreover, the reduction of area resulting from such a cooling process is probably inadequate to account for the observed degree of crumpling in the great mountain chains and need not be considered in this discussion.

The reduction of area due to the changing figure of the earth has been small during geological time so it is concluded that the compressional forces arising from systematic changes in density in the outer shell probably are responsible for the observed surface phenomena.[6]

Now the arcuate form of many mountain chains with overthrusting and wave-like folding on the convex side and the evident tendency of great mountain ranges to parallel adjacent coast lines has often been interpreted to mean that systems of tangential stress are applied to the continents, thus producing compression and crumpling on a grand scale. A system of tangential stresses competent to produce such crumpling has been shown in a companion paper [6] to be due to a non-uniform distribution of density in the earth's outer crust. This asymmetry is thought to have arisen fundamentally from the non-uniform cooling of the continental and oceanic hemispheres which must have amounted to some hundreds of degrees. The magnitude of the compressional stress was worked out assuming that systematic differences of density amounting to $2\frac{1}{2}$ per cent. were present as a result of non-uniform cooling and that isostatic equilibrium was everywhere maintained. It was found that the tangential acceler-

ations on the continents might easily approximate $7 \cdot 10^{-2}$ cm./sec.^{2}. From this estimate it was deduced that the compressional stress at the edge of a continent would approximate 10^8 dynes/cm.^{2}

A more recent examination of this matter has shown that the foregoing estimated stress may be correct or an underestimate according to the actual strength of the earth's crust down to a level of 1200 kilometers. If the strength is of the order of 10^8 dynes/cm.^{2} at all levels, then the original estimate is satisfactory, but if we follow the assumption made in the present paper, namely, that the crust below 100 kilometers may be plastically deformed with stresses, notably less than 10^9 dynes/cm.^{2}, the stress of the entire outer shell of appreciable strength may be concentrated in the stronger surface layers. For example, assume that tangential stresses of the order of 10^8 dynes/cm.^{2} are uniformly applied to a boundary region. The lower and weaker layers will be deformed and the plastic rock will be forced downward and thus relieve the stress. However, the material in the weaker layers may still retain an appreciable strength in shear which, of course, decreases with increasing depth. Therefore, points under the continent and remote, perhaps, from the region of maximum stress will be attached weakly at least to the outer and much stronger layers. Of course, adequate shear stresses may produce deformation but the magnitude of this shear in any region is proportional to the mean acceleration and to the mass below a selected level, so that the shear stress which must be supported in such a connected region increases as the surface of the earth is approached. Thus, since the strength of the crust also increases toward the surface, it seems possible that a tangential stress much larger than the average may be impressed in the outer layers. We have shown that the tangential acceleration in typical regions is $7 \cdot 10^{-2}$ cm./sec.^{2}, and since the density approximates 3 grams/cm.^{3}, the shear stress on a region at a depth of 100 kilometers due to a layer 1100 Km. thick below it is $S = adh$ or 2.3×10^7 dynes/cm.^{2} and at 500 Km. the stress is evidently 1.5×10^7. Because these stresses are nearly an order of magnitude less than the calculated compressional stresses on the boundary of the continent, it seems possible that the

entire force due to gravitational forces acting on the total thickness of the layer may be concentrated in the strong upper 100 kilometers of the outer shell. Such a concentration would imply a twelve-fold increase in the stress calculated before and it would therefore approximate 1.2×10^9 dynes/cm.^{2} A stress concentration of this magnitude acting at right angles to the continental boundaries should be entirely adequate to bring about the downward warping and compressional folding of the sedimented synclines considered in this paper and might quite readily produce some slow continental drift.[6]

The distribution of stresses at any instant in a continent subject to tangential gravitational forces is likely to be complicated. In the first place, as we have shown,[6] the stress is proportional to the difference in density of large regions of the outer shell. Therefore in the great majority of cases the stresses will be normal to the coast lines of the great continents and increase more or less regularly toward their boundary. Moreover, since the calculated stress is adequate, or at least nearly adequate, to produce failure of the resisting margin, then if the continent advances, first one localized region and then another will be subjected to enormous additional stress. Such localized stress will inevitably distort adjacent regions and zones of tension as well as compression will be set up. Thus, the crust may fail and large faults will be developed which may be filled with intrusive material. The distribution of tension due to any non-uniform distribution of compressional stress may be worked out, but since the pattern must constantly change as the continent advances it is probable that almost all continental regions which are normally under compression will be broken up and many dikes of basic intrusive rock may be expected.

In view of the assumption made in an earlier section, namely, that the tangential stresses increased more or less regularly with time, it is of interest to note that the magnitude of the average compressional stress in the outer shell according to Eq. 14 of reference (6) is proportional to the cube of its thickness. Moreover, since the thickness of the shell is proportional somehow to the amount of cooling, the difference in density of the continental and oceanic sectors increases

with increasing thickness of the shell. Now in the foregoing paragraphs it was assumed that in the course of development of a compressional fold into a geosyncline and a mountain, the stresses increased by a factor of about 3.3. Such an increase in stress corresponds evidently to a 35 per cent. increase in the thickness of the outer shell of appreciable strength. Since the development of a typical geosyncline requires a time at least comparable to the age of the earth, and this interval, in turn, approximates the time necessary to cool the outer shell, the foregoing estimate seems quite reasonable.

REMARKS.

It has been the main object of this paper to construct a quantitative theory of mountain building which is consistent with all the observed geological data of satisfactory reliability. The crucial difficulty, namely, to account for the deposition of great thicknesses of sediments in shallow water, has been met in a way which is consistent with many observational data and with fundamental physical laws.

The fundamental requirement of tangential stresses in the outer shell of a magnitude approximating 10^8 or 10^9 dynes/cm.^{2} and having a definite world-wide pattern is met satisfactorily, as we have shown, only by an earth having a marked asymmetry. Important geological inferences and a discussion of the mechanisms which are thought to be responsible for the observed earth features will be found in the author's companion paper.[6]

Although the calculations made in this paper are based on idealized structures and exact isostatic equilibrium assumed in every case, the results should be useful in analyzing actual mountainous regions and in guiding geological interpretation. The application of the present physical investigation to actual geological structures like the Cordillera will be considered elsewhere. It is sufficient to note here that the considered mechanisms seem adequate to describe completely the entire circum-Pacific mountain chain.

Because of the extraordinary complication suggested by geological development, we cannot be perfectly sure that all of the essential physics in mountain building has been included. But because the quantitative agreement with ob-

servation is satisfactory in the present paper, it is thought that a strong presumption is established in favor of the fundamental correctness and sufficiency of the adopted mechanisms.

The author is glad to acknowledge helpful comment and discussion of the present paper by Dr. R. W. Goranson of the Carnegie Institution of Washington.

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"Kemite."—In a release from the Mellon Institute, Pittsburgh, Pa., STUART M. PHELPS and EDWARD E. MARBAKER describe this new ceramic laboratory table top material. Materials commonly used for this purpose have many disadvantages and the thought was to overcome them by a new product. Experimental work extending over a period of years resulted in the production of a porous, non-warping ceramic body that is impregnated with bituminous substances and then heated under special conditions to form coke in the pores. Unusually high resistance to thermal shock is imparted to the material because artificial cordierite, a mineral that has a very low thermal expansion, is used in making up the ceramic body. This material, called "Kemite," can be polished to velvety smoothness, possessing sufficient hardness to resist scratching and abrasion, ample structural and impact strength, denseness that prevents absorption of liquids, resistance to solvent action and chemical attack, and the ability to withstand perfectly the effect of rapid heating and cooling. Because Kemite can be fabricated in complicated shapes of large size without joints, uses can be found for it in other fields. When high resistance to thermal shock is not required, cordierite is not used in the ceramic body and ware made from the non-cordieritic body is called "Karcite." Either body can be impregnated with sulfur or resinous materials to produce ware for special uses. This material may be used in the chemical industry for vats and tanks, in the electrical industry for specially molded parts and in building construction industry for ornamental forms. The ceramic body that is the basis of Kemite is being subjected to trial for special purposes for which accurate molding and resistance to thermal shock are important.

R. H. O.

MERCURY VAPOR GRID-CONTROLLED TUBES; THEIR ELECTRIC CONTROL BY A PHASE-SHIFTING NETWORK.

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I. INTRODUCTION.

The commutation or voltage regulation of grid controlled vacuum tubes may be performed in different proposed ways. Among the most discussed are the following methods: polarization batteries, phase shifting, harmonic frequencies, impulse method. All these methods have their advantages and disadvantages. The most interesting application of grid commutation is the two-way conversion of currents, i.e. a.c. into d.c. and d.c. into a.c.

The difficulty in converting d.c. into a.c. lies in the fact that it requires the a.c. load to possess a leading power factor. To obliviate that, different circuits are proposed of a more or less complicated nature. The method of harmonic commutation is one of them,[1] but requires a special disposit if for the supply of the 3rd harmonic. Recently Thomson-Houston Co.[2] has developed a method using phase commutation, which by appropriate connections of apparatus allows a two-way conversion. The power factor being any desired value regardless of the nature of the load.

The method of commutation, whether in a.c.–d.c. or d.c.–a.c. conversion, in the case of phase commutation remains the same. A general circuit allowing the two-way commutation may be thus established.

While there exists in literature a great number of schemes of connections describing different methods for commutation, as well as the equations for the voltage-current relations of the main circuit, the action of the grid is described in the majority

[1] See "Harmonic Commutation for Thyratron Inverters and Rectifiers" by C. H. Willis, *G.E.R.*, Dec. 1932, p. 632.

[2] French Brevet No. 743,343.

of these propositions in terms of physical action only. Therefore in this paper it is proposed to establish equations giving the voltage-current relations of the apparatus involving the grid itself. The method of phase shifting being in the author's opinion of the greatest possibility, the equations are established for this method. A circle diagram giving the amplitude and phase of the grid potential in function of the element of regulation is derived and the theoretical results checked experimentally.

The general equations for an application are then established and experimentally checked.

1. Discharge in a Three-electrode Vacuum Tube in Presence of a Residual Gas.

In a high-vacuum three-electrode tube the flow of electrons between cathode and anode is controlled by the voltage applied to a grid. It differs from the tube in which some residual gas is left by the purity of the electron flow. The electron current in a high-vacuum tube must be very pure; the presence of a few positive ions spoils the space-charge limiting repulsions on which the whole action depends. The electron current in the tube with residual gas, on the other hand, must be a hundred per cent. impure; that is, it must have as many positive ions as it has electrons. Electron repulsions are so completely neutralized by the presence of the positive ions that 10,000 as many electrons can be crowded into the space; hence this type of a tube can carry larger currents with lower voltage drop-amperes instead of milliamperes, with a few volts, 10 to 18, between anode and cathode, instead of hundreds of volts.

2. Principle of Commutation.

In a vacuum tube composed of a cold anode, hot cathode and containing small amounts of some gas or vapor, the discharge begins when the configuration of the electrical field between the anode and cathode attains a certain value, such that electrons escape from the hot cathode in a sufficient number possessing sufficient kinetic energy. When the tube is a mercury arc rectifier, for instance, then, under normal conditions and anodique tension of 15 volts is necessary. When a third electrode, the grid, is introduced into the tube,

connected to a definite negative potential, then the discharge may be prevented even at an anodique tension of many Kv., because then the resulting field (due to the anode and grid) may not provide a sufficient kinetic energy to the electrons. This is especially so before the beginning of a discharge. Once the discharge is started, then the functioning of the grid is unoperative. From the ionized molecules, i.e. the charge carriers of the two signs (positive and negative ions) which fills the tube, the positive ions flow to the negative grid giving it a positive space charge, neutralizing the negative charge of the grid. An increase of the grid potential increases the thickness of the space charge, making a change in the direction of the discharge (commutation) unworkable. First it is necessary to provide for the disappearance (deionization) of the positive space charge of the grid. Thus the commutation process is only a process of time control for the ignition of the anodes. If the anode potential is an a.c. potential, then the process of deionization is produced automatically at each cycle, the current having a zero value at each discharge. But one may change the time of restarting during the interval of a period, and by its change, for each given time of restart, obtain a certain value of current.

Because the commutation of vacuum tubes containing a grid and some gas or vapor is different from a two electrode high-vacuum tube (pure electronic emission vs. space charge influence), the treatment of the former does not require the use of the different incremental characteristics used in the latter, such as "Durchgriff" etc. In general a grid controlled partially gas-filled three electrode tube is characterized by a curve of the "critical" grid potential as a function of the anode potential at which the tube ignites. Such a curve is shown in Fig. 1.

Of the different possible methods of commutation we shall consider here the commutation by A.C. grid potential of a constant amplitude and variable phase, of which the mechanism is illustrated in Diagram 2.

1. Anode potential, 2. the critical grid potential when the grid potential is at 3 then the firing starts at A, and the anode current is given by the curve a, when the grid potential is moved to the curve 4, then the firing starts at B and the

FIG. 1.

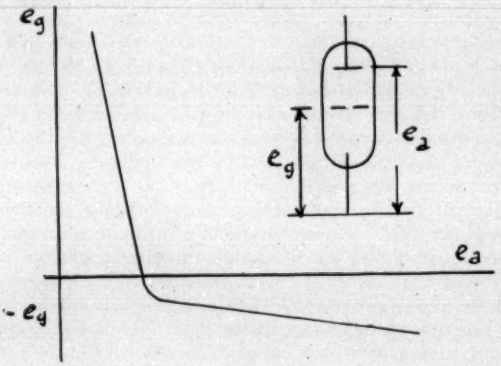
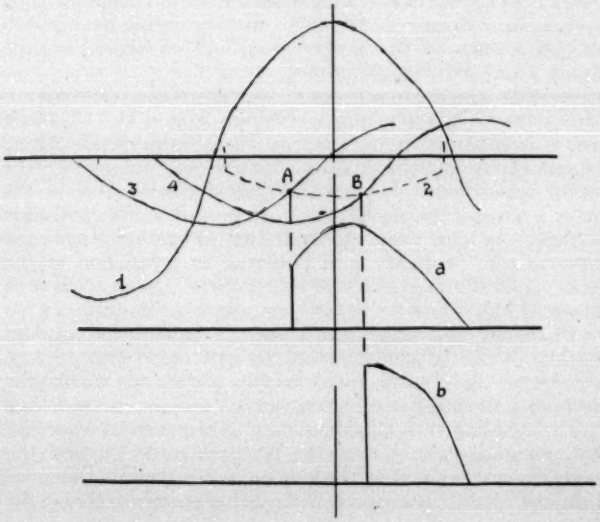


DIAGRAM 2.



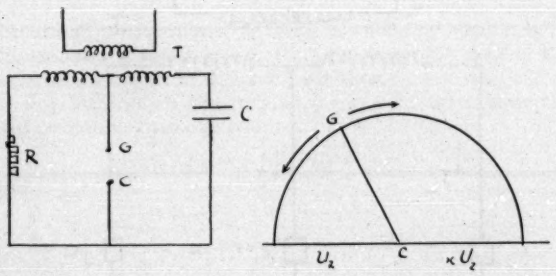
anode current is given by the curve *b*. It is thus possible by changing the phase angle between the anode and grid potentials to control the time of conduction of currents through the rectifier and thus to regulate its average value in any desired fashion.

II. THE PHASE SHIFTING BRIDGE REGULATOR.

The general discussion of the phase shifting method of regulation is considerably simplified for the simple case of single phase a.c. rectification. The scheme of connections for such a case is shown in the simple diagram of Fig. 3.

For the more complex case of polyphase rectification different combinations of the above single phase scheme are

FIG. 3.



available; their general operation is similar to the single phase case. The principle of the single phase rectification is illustrated in Fig. 3*a* and its corresponding vector diagram 3*b*. The secondary of the transformer *T* with the tap *G* is closed through a resistance *R* and a condenser *C*. The transformer potential *U* is divided into the voltage drop U_r through the resistance, and U_z through the condenser. These two voltages are at right angles to each other, thus the locus of the vector of the potential between the points *G* and *C* is half a circle with *U* as diameter. In the majority of proposed schemes of connections the phase shifter is loaded by the apparent resistances between *G* and *C*.

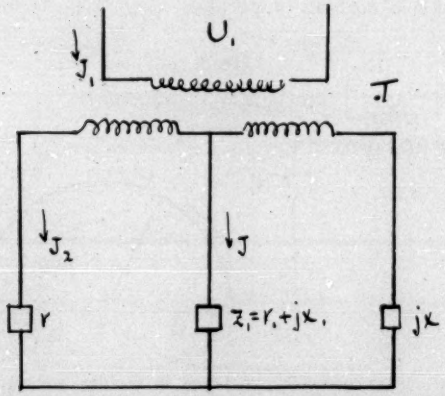
Because of this load, the potential GC is changed in amplitude as well as in phase angle. The variation in the amplitude depends on the position of the tap G.

1. General Equations.

We shall now derive the equations of the behavior of a bridge phase shifter connected and loaded as shown in Diagram 4.

The secondary winding of the bridge transformer (or potential dividing transformer) divides the secondary potential through the tap at G into two parts U_2 and kU_2.

FIG. 4.



These two potentials are closed, one through a pure resistance r and the other through a pure reactance jx. The potential of the branch GC of the bridge is the shifting potential which regulates the commutation of the tube. This branch has an apparent impedance $Z_1 = r_1 + jx_1$. In the above x and x_1 for generality may represent either an inductive (wl) or capacitive $(-1/wc)$ reactance.

Then the following equations may be written:

$$U_2 = J_2 r - J(r_1 + jx_1), \tag{1a}$$

$$kU_2 = J(r_1 + jx_1) + (J + J_2)jx. \tag{1b}$$

Multiplying (1a) by jx and (1b) by r and subtracting we obtain

$$U_2(kr - jx) = J(rr_1 - xx_1 + j(rx_1 + rx + r_1x)).$$

Introducing the following notation:

$$A = rr_1 - xx_1 \qquad \text{and} \qquad B = rx_1 + rx + r_1x$$

we obtain $J/U_2 = (kr - jx)/(A + jB)$ and

$$I/E_2 = ((k^2r^2 + x^2)/(A^2 + B^2))^{1/2}. \tag{2}$$

The tangent of the angle between J and U_2 is evaluated by equation (3)

$$\begin{aligned}\tan \phi &= -(xA + krB)/(krA - xB) \\ &= (kr^2(x + x_1) + x^2x_1 - rr_1x(1 + k))/(kr^2r_1 \\ &\qquad - x^2(r + r_1) - rxx_1(1 - k)).\end{aligned} \tag{3}$$

Equation (3) determines the range in which the phase may be shifted; this is found by inserting the extreme values of the regulation element in the above equation.

For instance, if the resistance r is used to regulate the grid potential, then one obtains:

$$\tan \phi_{r=0} = (x^2x_1/x^2r_1) = -x_1/r_1.$$

and

$$\tan \phi_{r=\infty} = -(x + x_1)/r_1.$$

By varying the reactance x, one obtains:

$$\tan \phi_{x=0} = -x_1/r_1$$

and

$$\tan \phi_{x=\infty} = -x_1/(r + r_1).$$

Thus the extreme values of the phase angle are independent of k, the proportionality factor of the bridge transformer's secondary subdivision, while the intermediate points do depend on the value of k.

2. Determination of the Conditions under Which the Amplitude of the Voltage Regulator Remains Constant.

We proceed to determine the conditions under which the current J and therefore the grid potential (through the impedance Z_1) remain constant for all angles of phase. We shall consider two cases.

Case (1). The grid impedance is a pure reactance, i.e. $Z_1 = jx_1$, and the voltage regulation is obtained by the variation of the resistance r. Then from eq. (2) it follows:

$$I/E_2 = ((kr^2 + x^2)/(x^2x_1^2 + r^2(x + x_1)^2))^{1/2}. \qquad (4)$$

This expression is independent of r, when [3]

$$k^2/(x + x_1)^2 = x^2/x^2x_1^2 \qquad \text{or} \qquad k = (x + x_1)/x_1.$$

This value of k modifies eq. (4) as

$$I/E_2 = 1/x_1. \qquad (4a)$$

Also eq. (3) becomes

$$\tan \phi = (r^2(x + x_1)^2 - x^2x_1^2)/(2rxx_1^2 + 2rx^2x_1).$$

The extreme values of the phase angle are in this case: for $r = 0$

$$\tan \phi_{r=0} = -\infty,$$

and for $r = \infty$

$$\tan \phi_{r=\infty} = +\infty.$$

Case (2a). The grid impedance is a pure resistance, i.e. $Z_1 = r_1$, and the voltage regulation is obtained by the variation of the reactance x. Then from eq. (2) it follows:

$$I/E_2 = ((k^2r^2 + x^2)/(x^2(r + r_1)^2 + r^2r_1^2))^{1/2}. \qquad (5)$$

This expression will be independent of x, when

$$k^2/r_1^2 = 1/(r + r_1)^2,$$

or

$$k = r_1/(r_1 + r).$$

If this value of k is inserted in eq. (5) we obtain

$$I/E_2 = 1/(r + r_1) \qquad (5a)$$

and eq. (3) becomes

$$\tan \phi = 2rr_1x(r + r_1)/(x^2(r + r_1)^2 - r^2r_1^2).$$

The extreme values of the phase angle are then in this case:

$$\tan \phi_{x=0} = -0 \qquad \text{and} \qquad \tan \phi_{x=\infty} = +0.$$

[3] For eq. (4) to be independent of r it is necessary that in the expression $(k^2r^2 + x^2)/(x^2x_1^2 + (x + x_1)^2r^2) = y$, the coefficients of r of the same power be equal, or $k^2/(x_1 + x)^2 = x^2/x_1^2x^2 = y$.

The use of reactances for voltage regulation introduces in practical applications certain difficulties. Especially for great ranges of regulation when the corresponding size of the element and its gradual subdivision involves great technical difficulties. It is therefore of interest to investigate a variant of this case, by using r as the element of regulation.

Case (2b). As in the case above $x_1 = 0$ and the grid impedance is a pure resistance $Z_1 = r_1$. The voltage regulation is obtained by the variation of the bridge resistance r. In a manner similar to the one used, for eq. (5) to be independent of r it is necessary that

$$k^2/(r_1^2 + x^2) = 1/r^2 \qquad \text{or} \qquad k = (r_1^2 + x^2)^{1/2}/r.$$

Then eq. (5) becomes

$$I/E_2 = ((r^2(r_1^2 + x^2)/r_1^2 + x^2)/(r^2r_1^2 + x^2(r_1 + r_1)^2))^{1/2} \qquad (5b)$$

and the value of the expression I/E_2 is the same when $r = 0$ and $r = \infty$ but varies for all intermediary values of r. Inserting the value of k derived from eq. (2), i.e.

$$k = ((r_1^2 + x^2))^{1/2}/r,$$

we obtain

$$\begin{aligned} I/E_2 &= ((r^2(r_1^2 + x^2)/r_1^2 + x^2)/((rr_1 - xx_1)^2 \\ &\qquad + (xr + xr_1 + rx_1)^2))^{1/2} \\ &= 1/r_1((r_1^2x^2 + r^2(r_1^2 + x^2))/(r^2(r_1^2 + x^2) \\ &\qquad + r_1^2x^2 + 2rr_1x^2))^{1/2}. \end{aligned} \qquad (6)$$

The minimum of I/E_2 in function of r is derived in the usual manner of finding the minimum of a function of one variable, or

$$\frac{d}{dr}(I/E_2) = \frac{d}{dr}(1/r_1((r_1^2x^2 + r^2(r_1^2 + x^2))/(r^2(r_1^2 + x^2) + r_1^2x^2 + 2rr_1x^2))^{1/2})$$

and from the above in the usual manner, equating the above to zero, we obtain

$$r = r_1x/(r_1^2 + x^2)^{1/2}.$$

On substitution of the above value of r in eq. (6) we arrive at the expression for minimum of I/E_2, which by introduction of

a new variable $v = x/r_1$ is given by

$$(I/E_2) \text{ min.} = 1/r_1(1/(1 + v/(1 + v^2)^{1/2})^{1/2}.$$

An examination of the above equation shows that the smaller is r the more nearly constant the potential remains. The power of the bridge potential transformer increases with smaller values of v. For the general case when $Z_1 = r + jx_1$ and neither r_1 nor x_1 is equal to zero, there is no value of k, which permits one to obtain a constant current J and thus a constant amplitude grid potential.

TABLE I.

r.	$r_1(I/E_2)$ min.
0.1	0.955
0.5	0.833
1.0	0.773

3. The Bridge (Potential Dividing) Transformer.

As shown in Fig. 4 the currents in the secondary winding of the transformer are J_2 and $J_2 + J$. From eqs. (1a) and (1b) we obtain

$$J_2/U_2 = (r_1(1 + k) + j(x_1(1 + k) + x))/((rr_1 - xx_1) + j(rx_1 + rx + r_1x)). \quad (7)$$

For the evaluation of the bridge transformer we are interested in the maximum value of (I_2/E_2).

If r is used as the regulation element, then from eq. (7) we derive:

$$I_2/E_2 = ((r_1^2(1 + k)^2 + (x + (1 + k)x_1)^2)/(A^2 + B^2))^{1/2},$$

where A and B are as defined on page 61.

Writing $\frac{d}{dr}(I_2/E_2)$, equating the so derived expression to zero and solving for r, we obtain

$$2r(r_1^2 + x_1^2 + x^2 + 2xx_1) = -2r_1x^2$$

or

$$r = r_1x^2/(r_1^2 + x_1^2 + x^2 + 2xx_1)$$

and

$$(I_2/E_2) \text{ max.} = ((r_1^2(1 + k)^2 + (x_1(1 + k) + x)^2)/(r_1^2 + x_1^2)x^2)^{1/2}. \quad (8)$$

Discussion.

Case 1. The grid impedance is a pure reactance, $r_1 = 0$ and $z_1 = jx_1$. In this case, as shown above the value of k is given in the case of a constant amplitude of the regulated voltage as $k = (x + x_1)/x_1$, and from eq. (8) we obtain:

$$(I_2/E_2) \text{ max.} = 2(x + x_1)/xx_1.$$

Case 2. The grid impedance is a pure resistance. $z_1 = r_1$, eq. (8), becomes

$$(I_2/E_2) \text{ max.} = (x^2 + r_1^2(1 + k)^2)^{1/2}/r_1x.$$

The other secondary transformer current, i.e. $I_2 + I$ is obtained in a similar manner, i.e.,

Case 1. $r_1 = 0$, $z_1 = jx_1$

$$((I_2 + I)/E_2) \text{ max.} = (2x + x_1)/xx_1.$$

Case 2a. $x_1 = 0$, $z_1 = r_1$; r—element of regulation

$$((I_2 + I)/E_2) \text{ max.} = (1 + k)/x.$$

Case 2b. $x_1 = 0$, $z_1 = r_1$; x—element of regulation

$$(I_1/E_2) \text{ max.} = ((r_1^2(1 + k)^2 + x^2)/(r^2r_1^2 + x^2(r + r_1)^2)^{1/2},$$

$$((I_1 + I)/E_2) \text{ max.} = (r_1(1 + k) + kr)/(r^2r_1^2 + x^2(r_1 + r)^2)^{1/2}.$$

Neglecting the magnetizing current, and the voltage drop in the transformer, the primary current J_p is evaluated from

$$J = U_2J_2/U_1 + kU_2(J_2 + J)/U_1$$
$$= (U_2^2/U_1)(J_2(k + 1)/U_2 + kJ/U_2)$$

for $r = 0$ and $r_1 = 0$.

$$I_1 \text{ max.} = (E_2^2/E_1)((x + x_1(1 + k)^2)/xx_1)$$

for $r = 0$ and $x_1 = 0$.

$$I_1 \text{ max.} = (E_2^2/E_1)((k^2x^2 + r_1^2(1 + k)^4)^{1/2}/xr_1).$$

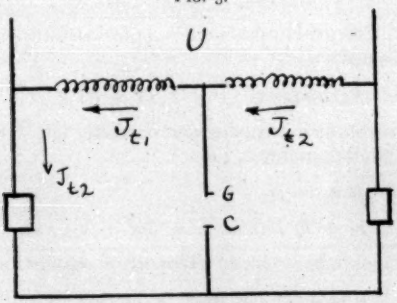
When the element of regulation is x and $x_1 = 0$, then

$$I_1 \text{ max.} = (E_2^2/E_1)(((k^2r + r_1(1 + k)^2)^2 + x^2)/(r^2r_1^2 + x^2(r + r_1)^2)^{1/2}).$$

The division of the potential of the phase shifter transformer may also be obtained by the use of an auto-transformer. The connections for this are shown in Fig. 5.

In some cases an auto-transformer may be used as a phase shifting transformer.

FIG. 5.



For this special case the currents are, when $r = 0$ and $r_1 = 0$,

$$J_{t1} = J_p - J_1 \qquad \text{and} \qquad J_{t2} = J_p - (J_1 + J),$$

$$I_{t1} = \frac{k}{x}\frac{E_p}{(1+k)^2}, \qquad I_{t2} = \frac{1}{x}\frac{E_p}{(1+k)^2}.$$

4. The Circle Diagram of Grid Potential in Function of an Element of Regulation.

Because it is easier to regulate the grid potential by varying a resistance and because in the most usual case the reactance of the tube may be neglected in comparison with its resistance, only the case when the grid impedance is considered as a pure resistance and the regulation is obtained by the variation of the bridge resistance, is treated. The other cases may be treated somewhat similarly. If it is desired to know the behavior of the grid potential for each value of the regulation element, and therefore for each corresponding value of the plate current then the following circle diagram giving the locus of that potential as a function of the value of r may be obtained as follows:

The grid potential is given by $U = Jr_1$. This may be

rewritten as $U = \underset{U_2}{J}\, U_2 r_1$ and with the use of eq. (2) this may be written as

$$U = U_2 r_1 (kr - jx)/(rr_1 + j(rx + r_1 x)) \qquad (9a)$$

and

$$E = E_2 r_1 (((k^2 r^2 + x^2)/(r^2 r_1^2 + x(r + r_1)^2))^{1/2}. \qquad (9b)$$

From eqs. (9) the phase angle between U_2 and U is given by the equation

$$\tan \phi^1 = (-rr_1 x(1 + k) - kr^2 x)/(kr^2 x - x^2(r + r_1)).$$

Eq. (9a) determines the locus of E, which is a circle. The proof follows: Eq. (9a) may be rewritten as [4]

$$U + JU^1 = r(U_2 + JU_2^1)k(r - jx)/rr_1 + j(rx + r_1 x))$$

or developing the above equation, we obtain

$$rr_1 U - (rx + r_1 x)U^1 + j((rx + r_1 x)U_1 + rr_1 U^1) = krr_1 U_2 + kr_1 x U_2^1 + jk(rr_1 U_2^1 - r_1 x U_2).$$

Equating real to real and imaginaries to imaginaries on both sides of the equation, it follows:

$$rr_1 U - (rx + r_1 x)U^1 = k(rr_1 U_2 + r_1 x U_2^1),$$
$$(rx + r_1 x)U_1 + rx U_1^1 = k(rr_1 U_2^1 - r_1 x U U_2).$$

Solving the two above equations each for r and equating the results, the following equation is derived:

$$x^2 r_1 U U^1 + kr_1 b^2 U U_2^1 + x r_1^2 U^{12} + kx r_1^2 U^1 U_2^1 - kr_1^2 x U^1 U_2^1 - k^2 r_1^2 x U_2^1 + x r_1^2 U^2 + kx r_1^2 U U_2 - r_1 x^2 U U^1 - kr_1 x^2 U^1 U_2^1 - kx r_1^2 U U_2 - k^2 r_1^2 x U_2^2 = 0$$

which reduces to

$$(r_1^2 x)U^2 + (r_1^2 x)U^{12} + (kr_1 x^2 U_2^1)U + (kr_1 X^2 U_2)U^1 - kr_1^2 x(U_2^2 - U_2^{12}) = 0.$$

The radius of this circle is given by

$$= (U_2^{12}(kx^2/2r_1 x)^2 + U_2^2(kx/2r_1)^2 + (U_2^2 + U_2^{12})^2(k/2)^2)^{1/2}.$$

The center is at the point

$$-U_2^1 \frac{k}{2}\frac{x}{r_1}, \qquad -U_2 \frac{kx}{2r_1}.$$

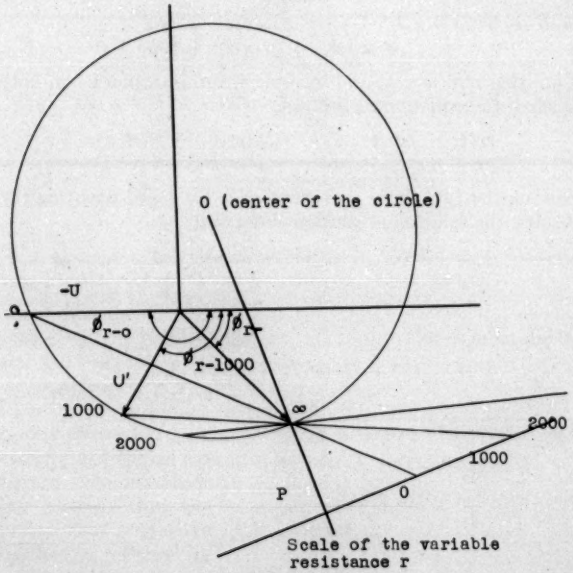
[4] See for the general theory on locus, Prof. Malti's "Circuit Analysis," Ch. 9.

Knowing the radius of the circle, as well as its coördinates the diagram for a particular case is drawn as follows: Let, for instance, the constants have the following values, $x_1 = 2000$, $x = 1000$, then $k = (x^2 + r_1^2)^{1/2}/r = 1.12$. The following table may be evaluated:

r.	E^1/E_2.	ϕ^1.
0	1	180°
1000	0,834	− 98°
∞	1	− 26° 40′

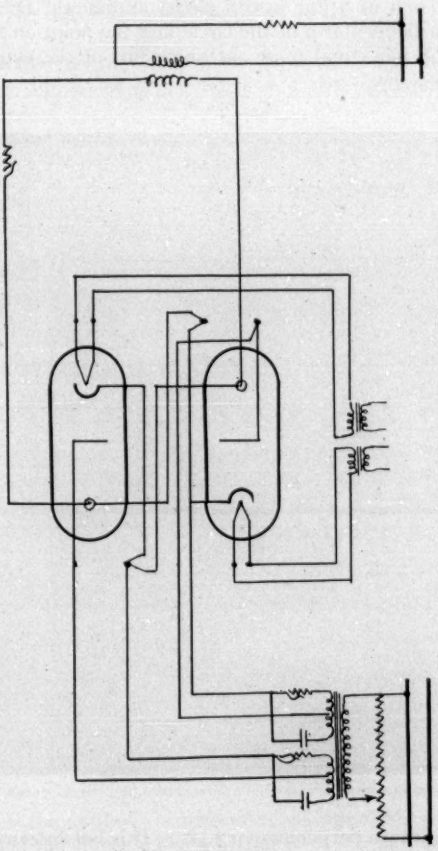
The circle diagram is constructed by finding the points on the circle for $r = 0$, 1000 and ∞, for which the values of E^1 and the corresponding phase angles ϕ^1 are given in the above table. This circle is shown in Fig. 6. The division of this

FIG. 6.



Circle diagram.

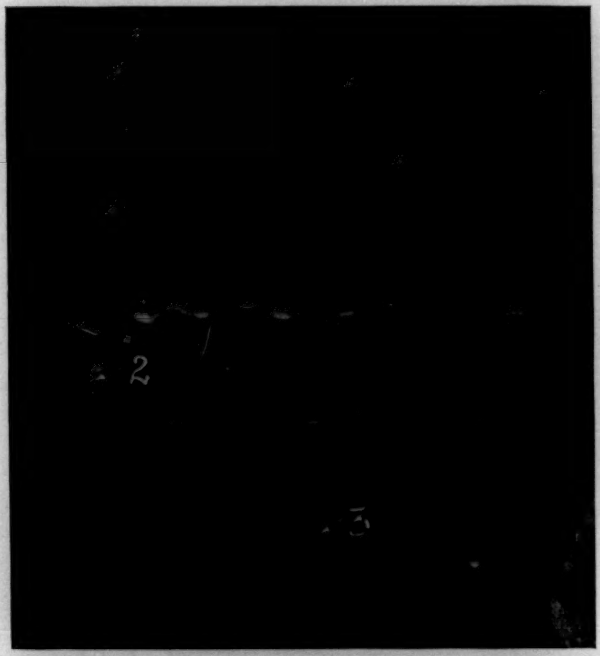
FIG. 7.



Connections diagram.

circle into different values of r is obtained, from the theory of inversion of a line into a circle, as follows: Draw a line through the center o of the circle and the point on the circle at which r is equal to ∞, at any point of the extension of

FIG. 8.



Oscillogram showing 1, supply potential, 2, grid potential and 3, plate current, of a fully opened tube.

this line draw a perpendicular to it. This line intersects with the line joining any value of r with the point $r = \infty$, at a definite point. Finding in this fashion the points on the line P corresponding to $r = 0$ and $r = 1000$, the distance between

these two points serves as a scale. Extending a line from a point on the line P through the point on the circle gives the value of r.

The above diagram determines the operation of the phase shifter for all values of r. For $r = 0$ the regulated (grid)

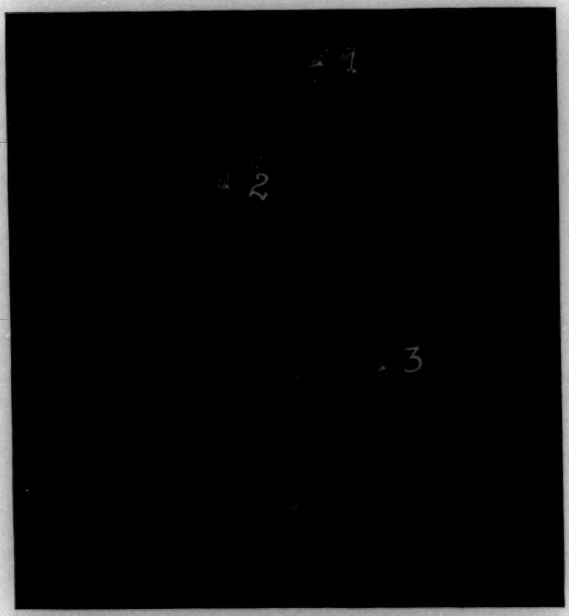
FIG. 9.

Oscillogram showing 1, supply potential, 2, grid potential and 3, plate current, of a half opened tube.

voltage is the applied (transformer) voltage, for $r = \infty$; the position of the vector is given by $\tan \phi = -x/r_1$. It may be noticed, that the voltage vector during the regulation goes through a minimum and then again reaches its original value. Next it may be observed that in the interval from $r = 0$ to

$r = 1000$ more than half of the regulation range is reached. It is thus important, that the resistance of the regulator be divided so as to obtain its change in certain definite steps, because its loading is different in different parts (due to the change in the voltage).

FIG. 10.



Oscillogram showing 1, supply potential, 2, grid potential and 3, plate current, of a nearly fully blocked tube.

Diagrams 8, 9 and 10 show the variation of the grid potential as a result of a varying bridge resistance. They were obtained for the same plate and primary transformer resistances and the same supply potential. For general wiring diagram see Fig. 7.

5. SUMMARY.

The results so far obtained may be now summarized as follows:

If a given potential is divided into two parts as shown in Diagram 3, one a voltage drop through a resistance and the other through a reactance, then the potential between the two points of division may be shifted through 180° when either the resistance or the reactance is varied in value from 0 to infinity. If this (shifting) potential applied across a pure reactance, and the phase shifting of the potential is obtained by varying a pure resistance, then its magnitude remains constant within the complete range of the phase interval, if the two parts of the potential are so arranged that the regulated (shifted) potential has the same amplitude when the regulation element has a value zero or infinity.

If, on the other hand, the regulated potential is across a pure resistance, then it remains constant in magnitude at all values of the regulation element, provided that the regulation is obtained by varying a pure reactance and if the two potentials are arranged as for the above case, i.e. the regulated potential has the same value for the two extreme values of the regulation element.

If the regulated potential is across a pure resistance, and its regulation is obtained by varying a pure resistance also, then the magnitude of the potential varies also. If the potential division is so made that the magnitude of the regulated potential is the same for the two extreme values of the regulation element, then it passes through a minimum, the value of which depends on the ratio of the constant reactance of the regulation network to the load resistance.

III. RECTIFIER TRANSFORMERS.

The general principles underlying the design of power transformers apply to rectifier transformers as well. The voltages applied to a rectifier transformer are sinusoidal, and the design of core and windings as regards induction, core losses, and volts per turn is the same as for other transformers.

Rectifier transformers differ from ordinary power transformers, used for transforming alternating-current power of one voltage to alternating-current power of another voltage, in respect to the character of the load of the secondary. The load of ordinary power and distribution transformers generally has a practically constant impedance per phase, over the whole cycle of alternating-current voltage, and draws a sinusoidal current from the transformer. Due to the valve action of the rectifier, each anode operates only during part of the cycle, so that each phase of the transformer secondary carries current only during part of the cycle.

As a result of this, the kilovolt-ampere rating of the transformer is higher than the kilowatt output from the rectifier, and, unlike ordinary power transformers, the kilovolt-ampere rating of the secondary winding is higher than that of the primary. The currents flowing in the windings of a rectifier transformer are irregular in shape, and not sinusoidal as for power transformers. The r.m.s. value of the current in each winding must be considered for the design of the windings and for the calculation of the copper losses. The reactive voltage drop in the transformer, produced by the non-sinusoidal currents, affects the wave shape of the secondary terminal voltage, and, therefore, the direct-current voltage characteristic of the rectifier.

The leakage reactance of a rectifier has a much greater effect on the regulation of the rectifier direct-current voltage than for an a.c. power transformer operating at the same power factor. For this reason it is usually desirable to have as low a leakage reactance as possible for rectifier transformers. The transformer must, however, be so designed as to be self-protecting in case of short circuit. On account of the unsymmetrical load imposed on the secondary windings, these windings must be so arranged and distributed with reference to the primary windings and to each other that balanced conditions exist with regard to the leakage reactance and stresses under normal as well as abnormal working conditions.

The secondary windings of rectifier transformers may also be subjected to high voltage surges, and for this reason the secondary windings and connections must be insulated for a higher voltage than required for power transformers of the same voltage rating.

Primary and Line Currents.

For rectifier transformers connected to a single-phase a.c. supply, the secondary currents are balanced at every instant by current in the primary, the condition being that the sum of m.m.f.s. on the transformer core is zero. For a polyphase rectifier transformer, the relation between secondary and primary currents depends on the type of primary connections used. In either case, the current relations are subject to the condition that the primary current cannot have any direct-current component, i.e. the average value of the current over a cycle must be equal to zero, because under steady-state conditions, a direct current cannot be induced in the primary winding.

(*To be continued*)

News of a New Blue.—(The *Du Pont Magazine*, Vol. XXXI, Nos. 1–2.) Color is all about us, yet we take color for granted, just as we do the rising and setting of the sun. We seldom pause to reflect upon what the world would be without color. Until the year 1856, man was dependent upon the vagaries of nature for his colors. In this year, a young English chemist, searching for a way to make synthetic quinine, stumbled upon a purple dye. Since then, man made colors grew to thousands of different shades and tints. The color of blue, however, did not progress as rapidly as other colors and up to now even the chemist had failed to produce a product of this type that gives complete satisfaction. But chemical research has developed a new blue pigment which is technically known as copper phythalocyanine. It is a du Pont product and its trade name is "Monastral" Fast Blue BS. There is a marked similarity between its chemical structure and the structures of hemoglobin and chlorophyll, the coloring matter of red corpuscles and green plants respectively. Yet "Monastral" Blue is far more stable than these two products of nature. Its uses are wide and varied for it fills a definite need for a blue capable of meeting the stringent requirements of many color consuming industries, both from the standpoint of application and fastness properties. It is extremely resistant to almost all of the severe color destroying agencies. It is particularly fast to light and to acids, alkalis and lime.

R. H. O.

THE STORY OF THE FRANKLIN INSTITUTE.

BY

SYDNEY L. WRIGHT, Ph.D.,

Associate Director, Medical Section and Director of Membership, The Franklin Institute.

IV. THE INSTITUTE TODAY.

The Franklin Institute is devoted now, as it has been for a hundred and thirteen years, to the increase of useful knowledge, to the encouragement of invention and discovery, and to the education of the public in the achievements of science and industry. Its very title has always indicated a desire to do honor to Benjamin Franklin. Recently, due to its position as the center of the Benjamin Franklin Memorial, all its activities are in a sense devoted to his honor, not only as a scientist, but as a man. It may truly be said that the Institute has the homely wisdom of Poor Richard ever before it and shares his desire to serve humanity.

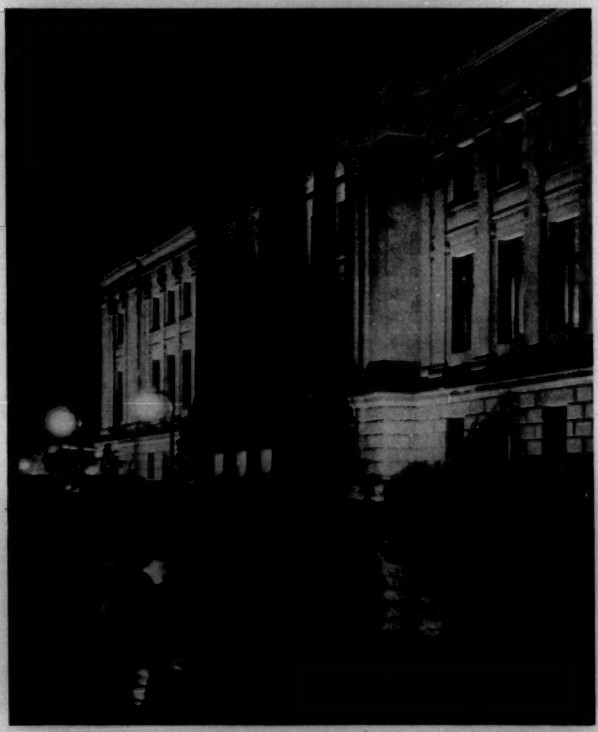
This service is rendered in a variety of ways and to widely differing groups. Industrial organizations, school children, college students and men and women engaged in a variety of technical or scientific pursuits, as well as the general public, are reached and aided by the Institute in several of its many activities. It is felt that a brief description reviewing its present organization and the manner in which it serves, will be of value and interest at the present time.

ORGANIZATION.

Control of the society rests in its members. The high position of the Institute ever since its foundation is evidence of the ability of its officers and of their devotion to the Institute. The President, the Vice-Presidents, the Treasurer, the Assistant Treasurer and the twenty-four members of the Board of Managers give their time and the benefit of their experience to the Institute without remuneration. In January 1937, Philip C. Staples was elected President, succeeding Nathan Hayward who had served faithfully for the eight

eventful preceding years. Mr. Staples, President of the Bell Telephone Company of Pennsylvania, brings to the Institute his experience in business and technology.

The administrative head of the Institute is its Secretary, Henry Butler Allen, who is responsible for carrying out, under



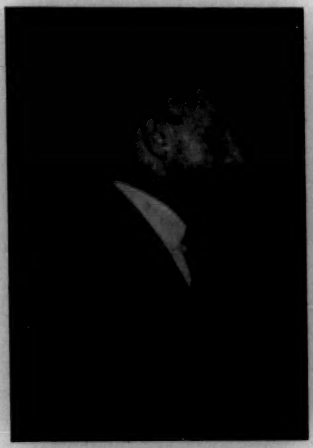
Wagner Schlesinger.

CHRISTMAS TIME AT THE FELS PLANETARIUM.

Beautifully designed inside and out, the building is maintained in a clean and attractive condition.

the President, the policies of the Board of Managers. The duties of the Secretary, therefore, cover the fields included under the several activities of the Institute.

The Memorial and Museum are under the supervision of the Director, who is at present one and the same person as the Secretary of the Institute. Under the Director, in direct



THE SECRETARY AND DIRECTOR.
Henry Butler Allen.

THE PRESIDENT.
Philip Clayton Staples.

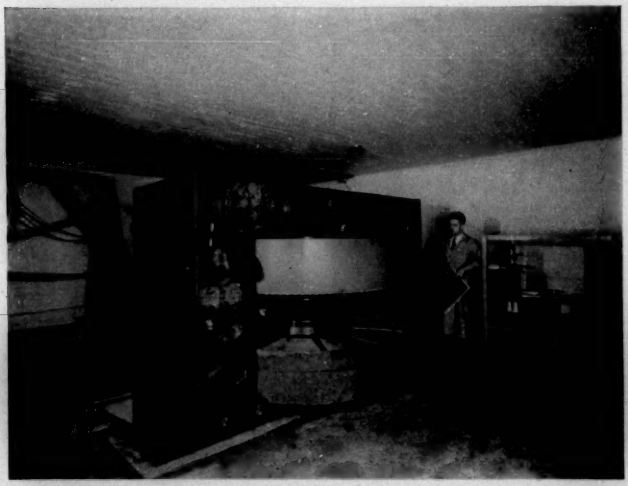
charge, is the Manager of Operations, William F. Jackson, Jr., who has assisting him the several associate directors, the technical staff, the building superintendent, the guides, the drafting room, the shops and other Museum departments.

The Library is in direct charge of Alfred Rigling, who is also Assistant Secretary and Assistant Editor of the JOURNAL.

The Committee on Science and the Arts has, as its Secretary, Dr. John Frazer, who resigned from his position at the University of Pennsylvania to replace the late Dr. George A. Hoadley, who served for many years, and to devote his full time to The Franklin Institute in which his father and grandfather had also been deeply interested.

A Department of Public Relations, under the charge of William A. A. Castellini, has been established to insure an adequate and proper presentation of the Institute to the public.

The accounts of the Institute are kept by the Controller, S. J. Clevenger, Jr.



THE CYCLOTRON.

The effect on living cells of the radioactive substances produced by this instrument is studied at the Biochemical Research Foundation.

Dr. James Barnes serves as Technical Adviser under the Secretary, and the Membership Department is under the charge of Dr. Sydney L. Wright.

The Bartol Research Foundation is under the Directorship of Dr. W. F. G. Swann and the Biochemical Research Foundation has, as its Director, Dr. Ellice McDonald.

Committees of the Institute and of the Board accept wide responsibility and give invaluable service in connection with the various activities.

ACTIVITIES.

These are many and varied in character and have been developed gradually as the need and the proper method of meeting it became evident. Some of these activities date from the first year of the Institute. Others are of much more recent origin and are under careful observation to determine their value and importance. The staff is aware of other services which would be of great value, but which cannot be provided at the present time with the limited funds of the Institute. Most of the more important sections of the

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IN THE LIBRARIAN'S OFFICE.

Mr. Rigling, Mr. Clevenger and Dr. Frazer meet to discuss a matter relating to the Committee on Science and the Arts.

Museum are in charge of an Associate Director or Technical Assistant who is responsible in a large degree for the planning and construction of the exhibits, the arrangement of the space and the manner of presentation to the public.

The Committee on Science and the Arts

Is formed of sixty-six members of the Institute. It reviews in great detail many of the advances in science and technology. During the past year, ten meetings of the entire

committee were held and fifty meetings were held by the various sub-committees. Fifteen awards were recommended to the Board and these were presented at the annual Medal Day exercises in May.

The Journal

Has maintained its high scientific standards and at the present time an edition of over twenty-six hundred copies is printed

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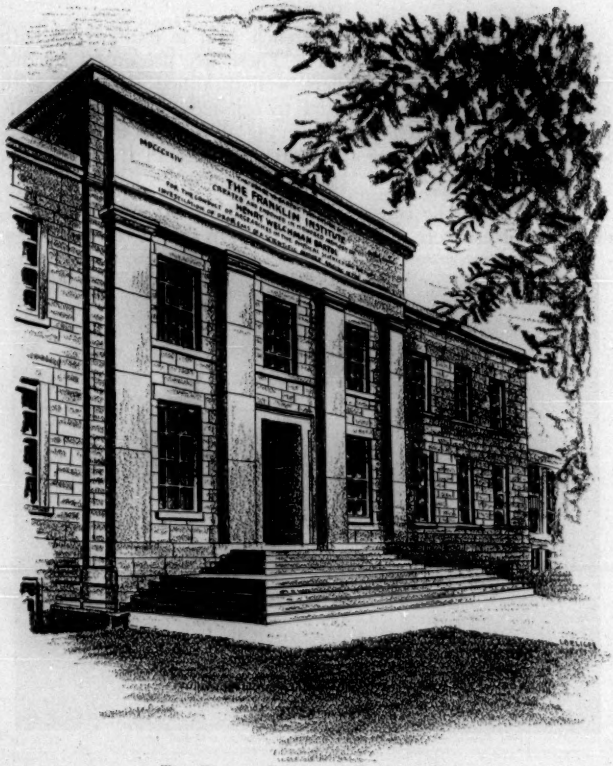
ELEMENTARY STUDENTS.

The complexity of the big locomotive and the many instruments to be observed by the engineer are not realized until they are examined at first hand. Then one can pull the throttle and start it moving—at a safe speed.

each month, of which nearly seven hundred copies go to forty-six foreign countries. It is estimated that nearly half of the entire issue is eventually deposited in libraries where it forms a useful and permanent contribution to scientific knowledge.

The Library

One of the most complete and useful in the fields of science which it covers, is particularly noted for the collection of



THE BARTOL RESEARCH FOUNDATION.
Research in fundamental physics is carried on by the Institute in this building at Swarthmore, Pa.

patent literature of this and other countries. In the past year many thousand visitors have made use of it, and several thousand requests for information have been received over the telephone. Appreciation of the service rendered by the Library is expressed by a number of firms which pay annually a Library Sustaining fee for its support and to secure its privileges for their employees.

LECTURES

Are given about twenty times each year in connection with the meetings of the Institute. These are presented by distinguished scientists or others well qualified to speak with authority on a subject related to science or industry. The lectures are well attended, averaging about two hundred persons, and the papers are frequently printed in the JOURNAL.

Research

Is carried on in the Bartol Research Foundation and the Biochemical Research Foundation. From these laboratories,

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AAAAH!

The familiar sigh of pleasure and amazement when the stars are first projected on the dome of the Planetarium informs the lecturer of the approximate number of his listeners who have never seen the instrument before.

additions are constantly being made to scientific knowledge. The Institute also operates a seismograph. This is a private station run in coöperation with the United States Coast and Geodetic Survey. Other opportunities to contribute to scientific knowledge are seized as they arise. For the eclipse of the sun on June 8th, 1937, The Franklin Institute co-

operated with the Cook Observatory of Wynnewood, Pa., and with Princeton University in sending two astronomers to make observations from a boat in the Pacific Ocean. The equipment that could be used aboard ship was limited, but as there were only two places on the path of the eclipse where observations could be made from land, the possibility of bad weather at those points made this expedition important. Weather conditions were excellent, and although satisfactory records were also obtained from other stations, Mr. Stokley of The

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STARS IN THE DAY TIME.

Many thousands have had their first telescopic glimpse of celestial bodies through these instruments. The sun and moon, and the brighter planets and stars can be observed on clear afternoons.

Franklin Institute and Dr. Stewart of Princeton have the satisfaction of being the only astronomers for twelve hundred years who have watched an eclipse for more than seven consecutive minutes.

MUSEUM.

The Fels Planetarium is under the direction of James Stokley, who prepares, with the assistance of his staff, a new demonstration for each month, so that in the course of a year it is possible to illustrate a wide variety of phenomena as they

are observed from different places and in different ages. He is also in charge of the Astronomical Section where are shown astronomical and surveying instruments and methods of telling time. The telescopes, located on the roof, are available for use whenever weather conditions permit.

The Aviation Section has as its head C. Townsend Ludington. Here are displayed wind tunnels and other devices for explaining the principles of flight. Model airplanes and scale

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FRANKLIN THE PRINTER.

A shop containing equipment of that period, where the visitor can compare colonial and modern printing methods. Mr. Davis is working at an imposition stone. The press used at Ephrata, Pa. is seen in the foreground.

models of well known craft together with exhibits of airplane construction and engines serve to round out the exhibit. Interesting features are the training plane in which the visitor can actually operate the controls, and the bulletin board on which are posted accounts of the progress of important aeronautical developments or expeditions. Several airplanes of historic interest are displayed.

The Graphic Arts Section occupies three large exhibition halls. Under the direction of Russell L. Davis, much of the printing required by the Institute is conducted by craftsmen

working under the somewhat trying conditions of constant scrutiny by the public and frequent requests for information. The work of the Institute is accomplished, and the public has the educational advantage of watching the process from the intertype or monotype machine or setting of type, to the printing and trimming of the finished product. On the ground floor a model papermaking machine actually manufactures a roll of paper, while the second floor is largely devoted to methods of illustration and reproduction of the fine arts.

The Chemistry Section contains displays of interest in fundamental chemistry and in chemical industry. These exhibits, especially the former group, were mostly designed and constructed by the Institute, and the manner in which complicated chemical reactions can actually be set in operation by the public is due to the ingenuity of the Associate Director, Dr. Nichol H. Smith.

The Medical Section, under the direction of Dr. Sydney L. Wright, portrays some of the contributions of the fundamental science to medicine, surgery and dentistry. The present small exhibit is but a nucleus from which will grow a display of great value in the field of public health that will present the subject in a manner appropriate to the importance of the various medical sciences in the life of civilized man.

The Section of Marine Transportation, under the direction of Charles M. Gay, and with the coöperation of a group of interested advisers, is developing plans which will lead to a comprehensive display of the development, since the discovery of America, of water craft and of marine engineering, including safety devices. Many models are now on display and future additions to the collection will be made in accordance with the plans developed.

The Music Section, under the direction of Guy Marriner, deals with music not only as an art, but as the product of the combined efforts of musicians, instrument makers, scientists and inventors. In the reproduction and transmission of music, in some of the most recently developed instruments, and in the understanding of the physical nature of tone quality and harmony, the last named groups have been especially conspicious. It is felt that the exhibits in this section, now on

display and included in the plan of development, serve to supplement a musical education and will increase the appreciation of good music on the part of the public. This latter cause is aided by the coöperation of this section in arranging musical programs when needed in other departments, and by the popular lecture-recitals given by Mr. Marriner during the winter months.

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FUTURE ADMIRALS.

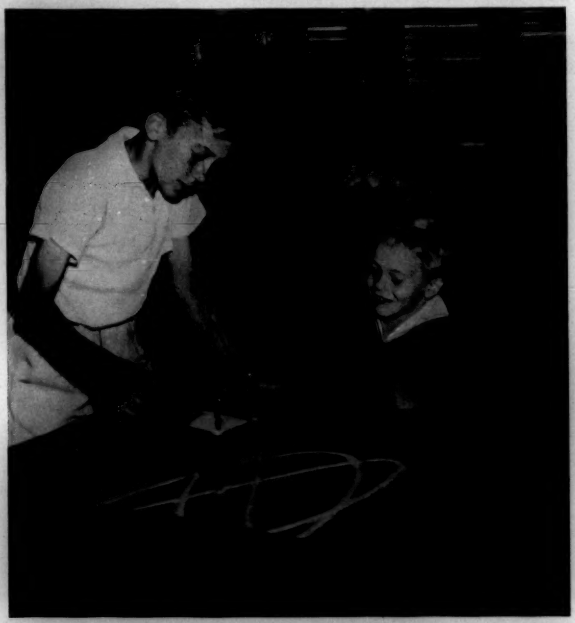
Students at the U. S. Naval Academy find it interesting to supplement their education at The Franklin Institute.

The Section of Electrical Communications is under the supervision of W. Parmelee West. Here are displayed the most modern types of telegraph, telephone, and radio, with exhibits illustrating the fundamental principles of each. The static machine used by Franklin, some of Faradays coils and replicas of Joseph Henry's inventions are also displayed in this section.

The Section of Illuminating Engineering discloses the methods of lighting since the settlement of America, and portrays the qualities and properties of various modern lights

which are of commercial or household interest. James D. Lee Jr. has charge of this exhibit.

The Physics Section was developed by Dr. James Barnes, and displays many of the fundamental principles of that



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THE COMPOSITION OF SIMPLE PERIODIC MOTION.
Unimpressed by the title, they see how simple it all is.

science with exhibits of an unusual and interesting nature. A portion of the section is darkened so that a number of the phenomena of radiation and some of the properties of light can be properly displayed.

Other Sections, that are under the general supervision of Mr. Jackson, Manager of Operations, include those of Land

Transportation, Fire Fighting and Prevention, Mechanics and Materials of Construction. The machinery used in the building for ventilation, air conditioning, elevators and other necessary functions are on display in the basement where they also can be observed in operation.

Special Shows. The continued and vital interest in the Museum on the part of the public is not due to the great variety and interest of the exhibits alone, although many visits would be required for a proper examination of the many

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A MUSEUM TALK.

The principles of streamlining are discussed by Ralph H. McClarren of the Aviation Section, who uses models of various shapes to show the flow of air.

sections. The staff is constantly alert for new principles to be explained or new methods of presentation. In consequence of this effort there is a continual change in the museum, and many special shows or demonstrations are introduced for limited periods of time. At least two of these are started each month, and it frequently happens that three or four special demonstrations are presented in a single day.

Some are held in the halls of the various sections and are presented to the public after criticism by members of the staff.

The effectiveness of the show in interesting and instructing the public is constantly studied and improvements are made where needed. Lecture-demonstrations which are likely to be of wide interest are held in the Lecture Hall. In many cases these include, together with new material, the most effective portions of the museum talks. Occasionally educational demonstrations prepared by industrial firms or other outside organizations are given space. Many of these have

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AN INDUSTRIAL EXHIBIT.

Mr. Jackson points out some of the desirable features of the exhibit of the Pennsylvania Salt Manufacturing Company to Mr. Castellini, Dr. Wright and Dr. Barnes. Note the push button which is used to start the action.

been of great interest and value, but few have left the Institute without some improvement in presentation. No corporation is likely to possess a staff of such diversified training, or of such experience in demonstrating science to the public, as has The Franklin Institute. The shop of the Institute and the experience of the staff are available to assist in the creation of scientific exhibits of educational value by anyone interested in the field of visual education.

Special shows during the past winter have included lectures and demonstrations on methods of communication from the cave man to the present time, photography under artificial light, electro-chemistry, flight in nature and human flight, the transparent woman, how to avoid blazes at home, motion pictures showing the manufacture of wire, five centuries of printing, an international exposition of design in outdoor advertising, photography since Daguerre and a number of others.

The Institute News. Such constantly changing centers of interest require that the public be notified of the time and location of these special shows. The Department of Public Relations prepares programs describing the activities of the day which are handed to each visitor on arrival and sends each month to every member of the Institute living in the United States a copy of "The Institute News" in which new exhibits, special shows and other activities are announced in advance.

Travelling Shows. The educational value of The Franklin Institute reaches beyond its doors in many ways—through its thousands of visitors, through the JOURNAL, and the publicating of members of the staff and of workers in the two research Foundations, through news dispatches and stories of the Institute and through activities devoted to the purpose of extending its educational influence beyond its walls.

The travelling Chemistry Show was developed through the stages of museum talks, and lecture hall demonstrations and the best features of a number of these were combined to make an entertainment which is completely portable and for which the only requirement is one 110 volt electric light socket. In the first eight months after its preparation, it gave one hundred and fifteen performances before nearly fifty thousand people in forty five cities and towns of which Detroit was the most distant. It has appeared before schools, in department stores, at clubs and at private parties. It has been presented in barns, at banquets and on the stages of theatres. The demonstration can be continued anywhere from thirty-five minutes to an hour and a half, and the first letter from a dissatisfied onlooker has yet to be received.

A similar show is being developed by the Aviation Section.

Portions of this are in satisfactory shape at the present time, but it is not completely worked out as yet, nor made portable, though a portion entitled "Flight in Nature" has already been shown in several cities.

A series of the briefer museum talks were given in a nearby private school. This was by way of experiment, but met

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OFF TO BALTIMORE!

Dr. Smith watches the truck of the Chemistry Show being loaded preparatory to leaving for a two weeks trip to Maryland.

with such success and enthusiasm that the venture will doubtless be repeated and become a part of the regular program of the Institute.

Other organizations make frequent use of the facilities and services of the Institute. Scientific and technical societies which are holding conventions in Philadelphia or which meet there regularly are apt to use it as a meeting place, and other groups use the Institute for an informal gathering arranged for the entertainment of visitors.

Several groups that are interested in the practical aspects

of certain fields and which include many young people in their membership, have developed a closer relationship with the Institute. The Junior Aero Club of Pennsylvania, the Rittenhouse Astronomical Society, the Ship Model Society and the Amateur Telescope Makers use the Institute as headquarters. A workshop for the last named is provided in the building. There mirrors can be ground and mounted under the supervision of members of the staff. Candidates

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AMATEUR TELESCOPE MAKERS.

Grinding a six inch mirror is good exercise. Testing and finishing it is interesting and instructive. By the time the mirror is mounted and in use, the operator has a keen appreciation of the value of the instrument and the underlying principles involved in its construction.

for certain merit badges and honors of the Boy Scouts come to the Institute for examination. A contest for prizes is held at the Institute for members of the Philadelphia Model Airplane Association. During the past winter a group has been meeting to study celestial navigation under the direction of Commander Weems, U.S.N. Ret. The Planetarium was found to be of great value in demonstrating the elements of this subject.

A radio broadcast entitled "Scientific Wonders" has been presented each week for more than two years through the coöperation of station WCAU. For the past year, except during the summer, "The Franklin Forum" has been the

title of a second series. Major Thomas Coulson has charge of these programs which have been commented upon by listeners from the United States and Canada and a few from Europe and South America. It is interesting that in many cases this is not the ordinary "fan mail." Requests for further details about the subject discussed are frequently received from responsible business concerns which have found the broadcast to be of value. On occasions of particular

Ralph H. McClarren.

BOY SCOUTS ARE INTENT.

And why not? With a wind tunnel at hand, the guide, W. H. Sheahan, is explaining why an airplane flies. Methods of demonstration were not so simple when he learned to fly in 1912.

interest broadcasts from the Institute have been carried on a national chain.

RESOURCES AND ATTENDANCE.

A measure of the success with which such an organization as The Franklin Institute conducts an educational program for the public benefit, is the number who come to the museum to receive the benefits offered. The Franklin Institute has attracted thousands of visitors in spite of the business depression and the existence of a charge for admission. In the

twelve months preceding this writing over 146,000 have visited the Planetarium and over 223,000 the Museum—a total of nearly 380,000. Many of these are Philadelphia school children admitted free of charge. The limiting factors in this case are the curriculum and the problem of transportatation. In the past forty months, however, winter and summer, an average monthly attendance of over 8000 children has been recorded.

Gladys Müller.

HIGH SCHOOL STUDENTS.

Brought by bus from Pottstown, Pa., they swarm into the Museum to see in operation experiments illustrating the principles of science mentioned in their textbooks.

The combined attendance at eight of the special shows mentioned above, for which counts are available, was a hundred and ten thousand, although two are still being demonstrated. More than five thousand attended the lecture recitals given by Mr. Marriner on Sunday afternoons, between three and four thousand came to the Institute meetings at which scientific papers were presented, more than six hundred were at the Christmas Week Lectures for Young

Gladys Müller.

FIRE ISLAND LIGHT.

These lenses, as they revolve, trace an intricate pattern of light on the walls and ceiling of The Franklin Institute. For years they served to guide mariners approaching the harbor of New York.

People and nearly thirty thousand were attracted by "The Skies of the First Christmas," which was a dramatization of the familiar story presented in the Planetarium during December through the joint efforts of the sections of Music and Astronomy.

This large attendance is an indication of the great popular interest in the exhibits and other activities of The Franklin Institute. It is vital to the Institute that this interest be

Gladys Müller.

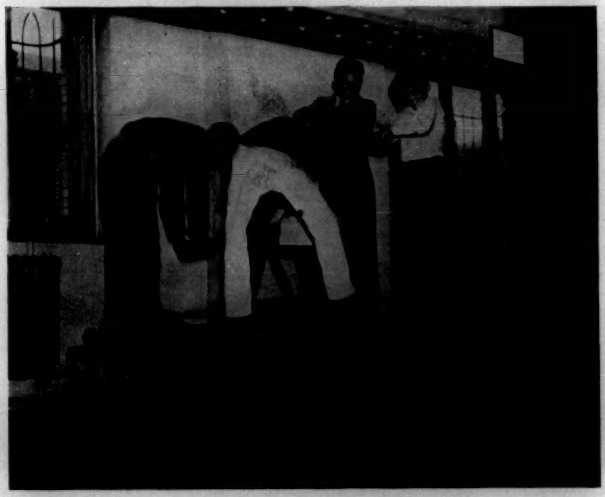
THE DAIRY DELL.

The lunch room is operated by the Dairy Council of Philadelphia for the benefit of visitors to the Museum. Trained dieticians are in attendance.

maintained, for the revenue derived from the small admission fee and the membership dues, amounts to more than forty per cent. of the entire income available for use outside of the research foundations. Income from the special and general endowment funds of the Institute amounts to a similar sum and the balance has to be made up of miscellaneous smaller items and the gifts of generous and philanthropic friends.

The Institute receives no help from taxation and is dependent entirely upon private resources. Although en-

dowment covering all expenses would probably be undesirable, it would be advantageous if the Institute were able to carry on all its ordinary activities without appealing to its friends except for new projects or unusual expenditures. Members of the staff would then be able to devote their time and energy to the services which they desire to perform and would not be constantly confronted with problems of a financial nature. Such important functions as the Library and the Institute



Gladys Müller.

AN AMBITION ACHIEVED.

How many have wished in vain to operate a hand car on the rails! These boys are enjoying the experience in The Franklin Institute.

Lectures should be completely endowed, so that there would be no possibility that these services will be in any way impaired. The funds now expended for these purposes would then be available to assist other departments of activity which are now handled on a basis of stringent economy.

The great number of operable exhibits in the Museum,

which require trained attention and an intelligent presentation to the public, makes the cost of operation of this museum far greater than that of an ordinary one of equal size. Besides the obvious consumption of light and power, a staff of expert

Gladys Müller.

FROM PULP TO PAPER.

An interested group watches the transformation taking place on the model Fourdrinier machine while Aiken Shearer, an experienced paper maker, explains the process. Later each one will receive a sample of the paper made.

mechanics must be readily available. Thirty-five guides, stationed throughout the building form the principle source of contact with the public. These men are well trained and educated and are competent to answer questions about the exhibits.

CONCLUSION.

This brief story of The Franklin Institute records some of the past accomplishments and present activities of the old organization. It is with a pardonable feeling of pride that members of the Institute view its record. The spirit of coöperation with which they and employees of the Institute approach its problems is a fortunate result of that feeling.

There is perhaps no joy so keen as seeing an awakened interest or understanding or a growing appreciation of the mysteries of science on the part of some individual drawn into the Institute perhaps from an idle curiosity.

What of the future? The Institute stands at the threshold of a new era. It has demonstrated once more the flexibility which enables it to meet the need of the age with an appropriate service. It is a leader in the practice of visual education as applied to science and it stands desirous and capable of increasing that service when the need and opportunity arises. The extent of its service is in the hands of its growing number of friends and will, therefore, doubtless be enlarged, for generosity and devotion to service are human qualities likely to endure.

Gladys Müller.

NINETY MILES FROM PHILADELPHIA

All the equipment, including the table, has been unloaded from the truck and the Chemistry Show is ready to start.

[*This completes "The Story of The Franklin Institute." It is expected that other articles, discussing certain activities of the Institute in more detail, will be prepared by Dr. Wright for publication in the Journal during the fall and winter.*]

COLLEGE LIBRARY

Artificial Vibration—A New Method of Dynamic Research.—(*Civil Engineering*, Vol. 7, No. 4.) An abstract of an unpublished paper by RUDOLF K. BERNHARD, a consulting engineer, states that artificial vibration consists essentially of inducing vibrations in the structure to be tested by means of an oscillator attached directly to it, and observing the energy consumed by the oscillator at various frequencies of vibration. All the dynamic properties of a structure can be determined by this method, including fatigue, endurance limit, damping ductility, damping capacity, period of resonance, and effect of time or aging on fatigue specimens. The method can be used on any size or type of structure from small laboratory specimens to bridges or battleships and can be used on any kind of structural material. The oscillator itself is a simple and compact machine. It consists of two equal masses, eccentrically supported, and arranged to rotate in opposite directions. This rotation sets up alternating forces which can be represented by a sine curve whose amplitude depends on the size of the masses, their eccentricity, and the speed of rotation. The latter factor can be regulated as desired. The oscillator may be driven by an electric motor. It can be built to operate at frequencies between 1 and 100 cycles per second, and is available in various sizes to produce centrifugal forces varying from 0.001 ton to 20 tons. Since in running a test, the force exerted by the oscillator is magnified by the vibration of the structure itself, particularly if the frequency is near the resonance period, pulsating forces of great magnitude can be set up. This method is economical. An oscillator is less expensive than any standard fatigue testing machine of equal capacity, is foolproof and easily operated.

R. H. O.

EXPLANATIONS.

BY

A. A. MERRILL.

One of the wisest statements ever made by Einstein was "Define your meanings in terms of operations."

When we say a thing is explained, just what do we mean? We may mean one or both of two things. We may mean that the question can be answered by the reaction of instruments, or we may mean that the ideas created in our mind by the language of the explanation, satisfies us personally so that we stop asking the question. This, I think, defines "Explanations" in terms of operations. Now there is a vast difference between these two explanations. The first is used, wherever possible by physical science and it can have a universal validity so far as humans are concerned, whereas the second can never have but a *local* validity. The four questions man wants answered are *where*, *when*, *why* and *how*. The answer to *where* can have a universal validity since that answer can be a picture, meaning the same thing to all human eyes. This is why the first human language was a picture language and why geometrical truths have such universal validity. The answer to *when* is always some privileged *where*, being either the position in space of some clock hand, or some astronomical body. The answers to *why* and *how* however, can never have a universal validity; their validity will always be *local*. A *why* which completely satisfies one mind may only confuse other minds. Just this fact is the explanation of changes in scientific theories, which theories are nothing but answers to *why* and *how*.

In the Victorian era the mind of even the best of physicists rested content with a mechanical model of reality, but not so now. Recent progress in physics has so altered experience that to satisfy a modern physicist you have to give him a differential equation. This "*why*" satisfactory to the physicist, is meaningless to most of us; that is, it has only a *local* validity.

Bürgenstock-Hammetschwand Mountain Lift.—(*Engineering*, Vol. CXLIII, No. 3715.) Bürgenstock is a well known summer resort on the south side of the Lake of Lucerne. It is situated on the Bürgenberg, a rocky peninsula which rises from the lake to a height of 3700 ft. In the year 1905 a promenade was cut in the rock face on the north side of the Bürgenberg for the benefit of guests staying at the Bürgenstock hotels, while a lift was erected from this promenade to the terrace on the highest point of the rock and giving access to the village of Hammetschwand. Owing to increased traffic the old lift proved inadequate and a new installation was put into operation in 1936. The lift car travels in a tower, built up of steel sections, which is connected to the rock face by a series of lattice ties. The upper tie forms a bridge connecting the top car-station with the terrace. The lower part of the tower is built into the rock and connects with a short vertical shaft which the car enters at the bottom of its travel. This connects with a horizontal heading giving access to the promenade. The tower is 6 ft. 6 in. square in cross section, and the car, which is built of aluminum alloy, can accommodate from ten to twelve passengers. Its total travel is 520 ft., and as it operates at a speed of 530 ft. a minute, it takes just under a minute to complete the journey. Ingenious winding gear and safety devices are employed.

R. H. O.

NOTES FROM THE BARTOL RESEARCH FOUNDATION.

THE DEFLECTION OF COSMIC-RAY CHARGED PARTICLES IN PASSING THROUGH MAGNETIZED IRON.*

BY

W. E. DANFORTH and W. F. G. SWANN.

Experiments are described in which the deflections suffered by cosmic-ray electrons in passing through the saturated iron core of an electromagnet are detected by means of Geiger counters. The observed effects are compared with the results of calculations in which we have used the energy distribution as found by Anderson and Neddermeyer with the cloud chamber. In this way we have found it possible to set limits to the effective magnetic vector within the iron. One of our experiments indicates a value lying between the induction B and $B/2$. The other points to the limits $3B/4$ and $B/4$. A theoretical discussion is included in which it is pointed out that all electrons of the same energy will not necessarily experience the same deflection but will show a statistical distribution of deflections with an arithmetic average corresponding to the induction B. The present type of experiment, however, does not give a true arithmetic average and would be expected to indicate, for the effective deflecting vector, a quantity less than B to an extent dependent upon the particular geometrical arrangement.

A THEORETICAL DISCUSSION OF THE DEVIATION OF HIGH ENERGY CHARGED PARTICLES IN PASSING THROUGH MAGNETIZED IRON.†

BY

W. F. G. SWANN.

The paper attempts a critical discussion of the situation pertaining to the magnetic field in a piece of magnetized

* An abstract of a paper published in full in the *Physical Review*, **49**, 582 (1936).

† An abstract of a paper published in full in the *Physical Review*, **49**, 574 (1936).

iron, particularly in relation to the deflecting force which it produces on high speed charged particles passing through it. On the Lorentzian theory the magnetic induction, B, is the average value of the true magnetic field H, averaged throughout the magnetic material. In this average, regions inside the magnetic entities responsible for the polarization make contributions which determine the whole difference between B and the ordinary macroscopically defined field, h, equal to $B - 4\pi I$. A study is made of the special case where the entities are rotating electrically charged spheres. If the entities are very small in volume the chance of a point electron missing all of them in its passage through a reasonably small length of the magnetized material is considerable. It appears that for such electrons as miss the entities the effective deflecting force is determined on the average by $h + 2\pi I$. The true average for all electrons passing through the material is determined by $B = h + 4\pi I$; but, that average is contributed to in appreciable amount by very few electrons which experience deflections much in excess of those determined by B. These considerations have important consequences in relation to the interpretation of experimental results.

THE ABSORPTION OF COSMIC-RAY SHOWERS IN LEAD.*

BY

C. G. MONTGOMERY and D. D. MONTGOMERY.

The absorption in lead of the shower rays which produce the bursts of cosmic-ray ionization is measured by two methods. The first method consists in observing the ionization produced above and below a lead absorber placed across the center of an ionization chamber; the second is to observe the probability that a burst of ionization in a chamber is accompanied by a simultaneous discharge of three Geiger-Müller counters over one of which has been placed an absorber. The results of the two methods are in good accord and may be stated in the form that the probability that a

* An abstract of a paper published in full in the *Physical Review*, 49, 705 (1936).

ray of a shower will penetrate a thickness of lead decreases linearly with the thickness, becoming zero at approximately 11 cm. The experiments serve to emphasize again the high energies that are involved in a large shower. The results are applied to observations on the effect of shielding on the ionization observed in the stratosphere.

ENERGY TRANSMISSION BY HIGH ENERGY ELECTRONS.*

BY

W. F. G. SWANN.

In a recent publication, J. R. Oppenheimer,[1] in seeking harmonization with the experimental facts concerning the absorption of cosmic-ray energy has been led to make a certain postulate. This postulate is to the effect that in considering the action of a high energy charged particle upon an electron of an atomic system, we must analyze the field of the high energy particle into harmonic waves according to the Fourier principle and then discard those frequencies which lie above a certain value ν_m whose order of magnitude is given by

$$2\pi\nu_m = v/\rho = 3m_0c^2v/2e^2, \qquad (1)$$

where ρ is the radius of the electron as given by classical dynamics, and is given in terms of the rest mass m_0 by the expression $\rho = 2e^2/3m_0c^2$. The velocity of the high energy particle is v, and is approximately equal to c, the velocity of light.

The purpose of the present paper is to show that the Oppenheimer postulate or its equivalent, is a direct consequence of the principles embodied in a generalization of wave mechanics which I published in 1933.[2]

The procedure adopted in my former publication is as

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* Reprinted from the *Physical Review*, 49, 829 (1936).

[1] Oppenheimer, *Phys. Rev.*, 47, 44 (1935).

[2] W. F. G. Swann, *Phys. Rev.*, 44, 943 (1933): also W. F. G. Swann, J. FRANK. INST., 217, 59 (1934).

follows:[3] According to the classical theory, with the neglect of so called radiation terms the acceleration $\ddot{s}$ of a particle of rest mass m_0 and charged e, is related to the electric field E the magnetic field H and the velocity of light c by the equation

$$m_0\ddot{s}/e = E + [\dot{s}H]/c. \tag{2}$$

It was recalled that, according to the theory of P. Ehrenfest and of E. H. Kennard, this equation could be represented wave mechanically in the sense that the centroid of the wave packet representing the moving charged particle moves according to (2), s being of course the acceleration in question. Remembering that the complete classical equation of motion of an electron is of the form

$$m_0(\ddot{s} - a_1\dddot{s} + a_2\ddddot{s}, \text{ etc.})/e = E + [\dot{s}H]/c, \tag{3}$$

the attempt was made to modify the wave-mechanical equation for ψ so as to cause it to lead to an equation of this type by principles analogous to those which through Kennard's theorem caused the ordinary wave-mechanical equation to lead to (2).

Writing (3) in the form

$$m_0P\ddot{s}/e = E + [\dot{s}H]/c, \tag{4}$$

where P is the operator defined by

$$P = 1 - a_1D + a_2D^2, \text{ etc.}, \tag{5}$$

D being written for d/dt, it was shown that the desired end could be secured by *replacing the scalar and vector potentials φ and U of the external field in the ordinary wave-mechanical equation by $P^{-1}\varphi$ and $P^{-1}U$.*

The particle whose equation of motion is under discussion is of course the electron whose ejection from the atom is being examined. Its velocity is small in contrast to that of the high energy electron responsible for the scalar and vector potentials φ, and U. Thus, it is possible to replace d/dt by

[3] For the present purpose I omit certain refinements contained in the original papers, and having to do with the magnetic field and with relativistic invariance of the equations. These matters are of only secondary importance in the present discussion.

$\partial/\partial t$ in P; and, in what follows, we shall regard D as meaning $\partial/\partial t$.

Now following the foregoing principles and taking, for simplicity, but not of necessity, as illustration, the case where we confine our attention to a classical Eq. (3) which has no more than two terms on the left-hand side, our corresponding operator P is $(1 - a_1 D)$. The wave-mechanical principle applied according to the Kennard method to produce (2) must now be modified by replacing φ and U by φ_1 and U_1, where

$$\varphi_1 = (1 - a_1 D)^{-1}\varphi, \qquad U_1 = (1 - a_1 D)^{-1} U. \tag{6}$$

Following Oppenheimer, we expand the field, or the field potentials, in the form of a Fourier integral.[4] Thus, for example,

$$\varphi = \int A_\nu e^{2\pi i \nu t} d\nu,$$

where it is to be understood that the real parts are to be taken. While (6) with a corresponding expression for U gives the expansion of the potentials, the effective potentials required by our theory are φ_1 and U_1. Thus,

$$\varphi_1 = \int \frac{A_\nu e^{2\pi i \nu t}}{1 - a_1 D} d\nu.$$

Now any function of D operating on $e^{2\pi i \nu t}$ may be replaced by the same function of $2\pi i\nu$ operating on $e^{2\pi i \nu t}$. Hence,

$$\varphi_1 = \text{Real part of} \int \frac{A_\nu e^{2\pi i \nu t}}{1 - 2\pi a_1 \nu i} d\nu,$$

[4] We only need expression of the integral in its dependence on the time; for, as the high energy particle passes by the electron to be ejected, the latter experiences, in view of the small velocity to which it is limited, variations in φ which are concerned only with the time variations of that function. The complete expression of the field involves a coefficient A_ν' of the form given by Oppenheimer,

$$A_\nu' = (2\pi^2 i e\nu/2\epsilon v^2) H_1^{(1)}(2\pi\nu p i/\epsilon v) e^{-2\pi i \nu z/v},$$

where v is the velocity of the high energy particle, $\epsilon = (1 - v^2/c^2)^{-1/2}$, where the high energy particle moves parallel to the axis of z, and the origin is at the electron to be ejected. p is the perpendicular distance from this electron to the path of the high energy particle, and $H_1^{(1)}$ is the Hankel function of the first kind.

i.e.,

$$\varphi_1 = \int \frac{A_\nu \cos(2\pi\nu t + \theta_\nu)d\nu}{(1 + 4\pi^2 a_1^2 \nu^2)^{1/2}}, \tag{7}$$

where

$$\tan \theta_\nu = 2\pi a_1 \nu.$$

When $4\pi^2 a_1^2 \nu^2$ becomes appreciably larger than unity the value given by (7) for the contribution of the corresponding frequency to φ_1 becomes small compared with the contribution of that frequency to φ. In other words, the frequencies which in Oppenheimer's theory are discarded, are here rendered *impotent by the natural consequences of the theory*. The theory thus provides a basis for the Oppenheimer conditions. Without such a basis the most that can be said concerning frequencies greater than the critical value is that the contributions which the ordinary theory would cause them to make to the story are probably wrong. This, alone, does not provide for the actual contributions being negligible, however.

On the classical theory, the quantity a_1 occurring in (3) and in subsequent equations would be equal to ρ/c, where ρ is the classical radius of the electron. Thus, the limiting frequency ν_m is given by

$$2\pi\nu_m = c/\rho = 3m_0c^3/2e^2. \tag{8}$$

Thus, since $v = c$ for high energy particles, we see by comparison with (1) that (7) not only reproduces the qualitative significance of the Oppenheimer conditions but provides for them quantitatively. It must be admitted, however, that it would be straining matters too much to hold to the magnitude of a_1 predicted by the classical theory, particularly as there is no direct experimental verification of the magnitude of a_1 as there is for the mass of the electron. The significant thing is that there is *some* value which is appropriate for this coefficient.

In conclusion, I wish to thank Dr. A. Bramley for calling the Oppenheimer conditions to my attention.

THE SPECIFIC IONIZATION OF COSMIC-RAY PARTICLES AS DETERMINED BY GEIGER-MÜLLER COUNTER EFFICIENCY.*

BY

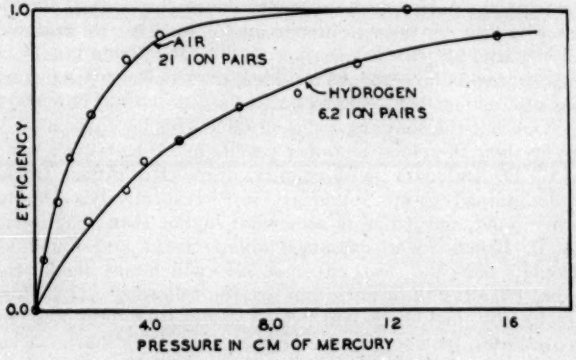
W. E. DANFORTH and W. E. RAMSEY.

The efficiency of a Geiger-Müller counter, used in a coincidence arrangement for cosmic-ray counting, is determined, at least in part, by the primary ionization along the path of the particles.

The results here reported seem to show that the efficiency is determined to such an extent by the primary ionization that measurements of the former may serve as a satisfactory means of determining the latter.

The points plotted in Fig. 1 show measured values of

FIG. 1.



Efficiency of counter as a function of pressure.

efficiency of air-filled and hydrogen-filled counters at different pressures of the counter gas. The smooth curves, however, are calculated on the basis of the indicated values of the primary specific ionization at N.T.P., 21 ion pairs per cm. for air, and 6.2 for hydrogen.

* Reprinted from the *Physical Review*, 49, 854 (1936).

In these calculations we have assumed that (1) the presence of one ion pair is sufficient for the initiation of a discharge, and (2) that negligibly few entities capable of exciting a discharge are ejected by the ray from the inner surface of the counter cylinder. The efficiency is then obtained by subtracting from unity the probability that a particle will traverse the counter without suffering an ionizing collision with a gas molecule. This probability, calculated from the Poisson law of the random sequence, involves the primary ionization and the distribution of path length within the counter.

The fact that the observed points lie satisfactorily about the calculated curves constitutes, in itself, a tentative verification of the two above-mentioned assumptions. Assumption (2) concerning the absence of wall effect has been studied in an independent experiment in which it was found that increasing the path length augments the efficiency to about the same extent as does a proportionate increase in pressure. The path length distribution problem in this auxiliary experiment is, however, so complex, that an exact calculation has not, as yet, been carried out.

One notices that the value obtained for air (21 $cm.^{-1}$) is lower than the cloud chamber results of G. L. Locher [1] and of C. D. Anderson [2] who counted droplets in diffuse tracks and obtained 36 $cm.^{-1}$ and 31 $cm.^{-1}$, respectively. On the other hand, our value is somewhat higher than that found by P. Kunze [3] who examined sharp tracks and found 19 droplets per cm. Two circumstances which may render the cloud chamber data uncertain are the following: (1) Diffuse tracks, by including ions which have been produced by secondaries, tend to give too high a value. (2) Sharp tracks, on account of random grouping and coalescing of droplets, tend to give too low a value. Kunze, however, applied a statistical correction to take care of random grouping. Our value for air is slightly lower than that found by E. J. Williams and Terroux [4] for a million-volt beta-particle in

[1] G. L. Locher, *Phys. Rev.*, **39**, 883 (1932); J. FRANK. INST., **217**, 39 (1934).
[2] C. D. Anderson, *Phys. Rev.*, **44**, 406 (1933).
[3] P. Kunze, *Zeits. f. Physik*, **83**, 1 (1933).
[4] E. J. Williams and Terroux, *Proc. Roy. Soc.*, **A126**, 289 (1930).

oxygen. For hydrogen however, our value is somewhat higher than that given by these authors (5.1 cm.$^{-1}$) for a million-volt beta-particle.

This method should be able to detect inhomogeneity of the radiation as regards specific ionization. Interpreted as homogeneous radiation, a distribution of specific ionization would, at low pressures, give an effective ionization equal to the average, but at high pressures the effective value would be less than the average. The present results show a rather surprising degree of homogeneity.

Counters operating at low efficiency form a means of discriminating between radiations of different primary ionizations. We have in progress an experiment designed to detect, by this means, the presence of low energy protons in the radiation—entities which should exist in measurable numbers if, as has been suggested by Compton and Bethe,[5] protons form an important component of the sea level radiation.

Measurements on different gases are now in progress in the hope that the results will be of use in the theoretical problem of ionization by high energy particles.

[5] A. H. Compton and H. A. Bethe, *Nature*, **134**, 734 (1934).

Weighing Problem Solved.—(*Chemical and Metallurgical Engineering*, Vol. 44, No. 3.) Probably the world's largest automatic liquid weighing scale was recently delivered to the Cuban Government for measuring cargoes of oil and gasoline. Satisfactory operation of this scale depends upon the oil-proof character and flexibility of vital rubber parts. Using Thiokol synthetic rubber, the Richardson Scale Co. was able to give an unconditional oil-proof guarantee to those parts. Of particular engineering interest is the use of this same plastic material for hose and valve seat disks. Cuba's big scale weighs 27,500 lb. of gasoline and oil each minute. To convey and weigh this amount accurately, the manufacturer was confronted with the problem of obtaining 12 in. hose which was exceedingly flexible as well as gasoline proof, as the liquids are conveyed from a feed pipe to two large floating tanks, and any resistance to the delicate movement of the weighing tanks mounted on scale levers, would seriously affect the accuracy of the scale. For the same reasons, leakless gasoline-proof rubber valve seat disks were designed and manufactured of Thiokol to permit and insure instantaneous stoppage of the liquid flow at exactly the proper moment. Tests revealed that the scale registered an error of less than 1/900 of 1 per cent.

R. H. O.

V.
19

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

THE IONOSPHERE AND RADIO TRANSMISSION.

The ionosphere consists of several layers of ionized or electrically conducting air from 60 to 300 miles above the earth. These layers act as reflectors for radio waves and make possible radio transmission over long distances.

Depending on the degree of ionization of a layer, there is an upper limit to the frequency which, when the waves are sent straight up, may be reflected from the layer. Radio waves of frequencies greater than this upper limit, or critical frequency, go completely through the layer and pass entirely out into space. The critical frequency for each layer varies with the hour of the day, season of the year, and also over a long period which seems to be associated with the 11-year sunspot cycle.

The air in these layers is ionized principally by ultraviolet light from the sun. Consequently, the critical frequency would be expected to be greater during the day than during the night and greater during the summer than during the winter. This is true for the lower layers, called E and F_1 layers. The uppermost or F_2 layer does not behave so regularly but attains its daily maximum critical frequency shortly after noon from October to March and about sunset from April to September. Strange as it may seem, the maximum in the former period is considerably greater than in the latter. After sunset the critical frequency decreases but does not fall to zero.

The use of a multifrequency automatic recorder has made it possible to secure hourly ionosphere records at the Bureau since May 1933. The recent adoption of the idea of multifrequency automatic recording and the principle of this recorder by the Carnegie Institution of Washington, British Radio Research Board, Australian Radio Research Board, and

* Communicated by the Director.

Harvard University will increase enormously the continuity and value of ionosphere data, which will in turn greatly increase our understanding of world-wide ionosphere and radio transmission conditions.

The average critical frequencies and heights of the several layers, as measured near Washington, D.C., for each hour of the day for each month from May 1934 to December 1936 inclusive are plotted in RP1001 by T. R. Gilliland, S. S. Kirby, N. Smith, and S. E. Reymer, which will be published in the June number of the *Journal of Research*.

The upper limits for frequencies which can be used for transmission over long distances are several times greater than, but proportional to, the critical frequencies. Therefore, higher frequencies may be used for transmission during the winter day than at any other time, and higher frequencies may be used during the summer evening than at any other time of the summer day. Also, because of the increased sunspot activity, higher frequencies could be used in 1936 than in 1933.

In addition to the critical frequencies, the heights of the layers and the amount of absorption of the radio waves play an important part in long-distance radio communication. The effects of these are discussed in the paper, and also two types of irregular disturbances of the ionosphere, which affect radio transmission.

COMPENSATION OF STRAIN GAGES FOR VIBRATION AND IMPACT.

The demand for adequate instruments for the measurement of strain in structures in vibration or under impact loads is well known. One of the difficulties encountered in trying to apply existing strain gages to the measurement of dynamic strain has arisen from deformations within the gage caused by the inertia of the gage itself when subjected to high accelerations. These deformations in general affect the reading of the gage and may cause serious errors. It has been customary in the past to decrease the mass and increase the rigidity of the gage in an effort to minimize these deformations, and correct for the small resulting error, if

necessary, by means of control experiments. It is, however, frequently not feasible to construct a gage sufficiently light and rigid to make this possible.

A new method developed by W. M. Bleakney, and presented in the *Journal* for June (RP1005), consists in so adjusting the ratio of stiffness to inertia of the parts of the gage that these deformations are compensating. The indication of the instrument may thus be made independent of any acceleration of the gage as a whole.

This method is being used in the design of an electromagnetic strain pick-up unit developed for the Navy Department for the recording of strains in airplanes during flight. The Massachusetts Institute of Technology and the Sperry Gyroscope Company are collaborating in this work. Further applications of the compensation principle will, no doubt, be found as improved instruments for measuring dynamic strains become available.

SOIL CORROSION TESTING.

In considering the subject of soil corrosion two major factors require special attention: (1) the influence of the nature of the metal, and (2) the influence of the soil. The chief metals which are exposed to the soil in practice are iron and steel, and lead, copper, and brass. Zinc also is used extensively in the form of a coating. Since by far the greatest amount of metal exposed to the soil is iron and steel, research in the field of soil corrosion has been concerned chiefly with these metals.

The sources of information on soil corrosion are: (1) examinations of pipe lines in service; (2) field tests of selected metals buried in selected soils; and (3) laboratory tests of metals in soils. Analysis of corrosion data from these sources has led to a procedure by which the corrosiveness of soils toward iron and steel can be determined.

A rational procedure for testing the corrosion of metals in soils must take into account: (1) the corrosiveness of the soil; (2) the relation between pit depth and area; and (3) the relation between pit depth and time. The corrosiveness of a soil toward ferrous metals may be determined in the laboratory

by measuring the voltage at various current densities of a cell in which the electrodes are steel and the electrolyte is the soil under test. The average current density corresponding to a range of voltage between 0 and 0.3 v. is expressed in terms of the average depth of the deepest pit to be expected on 1 ft.^{2} at the end of 12 years. By means of an equation in which the depth of the deepest pit on a given area is expressed as a function of (1) the corrosiveness of the soil; (2) the area exposed; and (3) the period of exposure, the estimated depth of pit may be calculated for the desired area and time. A general equation has been developed for estimating the number of holes, if any, in pipe as a function of the corrosiveness of the soil, time, and wall thickness.

Empirical equations in which the depth of pit is expressed as power functions of time and exposed area have, likewise, been worked out, and the constants of the equations have been related to the properties of the soil. The slope of the pit depth-time curve when plotted on logarithmic coördinates has been found to be determined largely by the aeration of the soil; the more poorly aerated soils produce a slope.

The precision and range of application of the suggested procedure for soil corrosion tests is limited by imperfect understanding of certain features of soil corrosion, such as the exact nature of the relation connecting pit depth with exposed area. Although the Bureau's present method is not sufficiently well established to be adopted generally, it does constitute a basis on which a standard method may be developed.

ACCELERATED WEATHERING TESTS OF MINERAL-SURFACED ASPHALT SHINGLES.

Recent tests by H. R. Snoke and B. E. Gallup of the Bureau's chemistry division, on mineral-surfaced, asphalt shingles, show that all of the types of failure encountered in long outdoor exposures to the most severe weather conditions can be produced by accelerated weathering in 7 months or less.

As reported in RP1002 in the *Journal of Research* for June, 39 samples of standard-weight shingles, from eight of

the larger manufacturers, and including the natural and artificial granular surfacing materials most commonly used, were included in the test. Except that the shingles submitted by one manufacturer contained no fine mineral filler in the asphalt coatings, and that those from another were low in percentage saturation of the felt, analyses of the samples showed no differences in composition differences in their behavior to weathering.

Outdoors exposures of these shingles normally require from 10 to 20 years to produce definite results, and consequently are of little value in predicting the behavior to weathering of materials currently available, since raw materials and manufacturing processes are constantly changing. The accelerated weathering test should therefore be of considerable value, not only to purchasers of large quantities of these materials, but to manufacturers, as a means of improving their products.

BEHAVIOR OF LEATHER IN THE OXYGEN BOMB.

The complete analysis of a sample of leather includes ten or more separate determinations, and the time required for the completion of such an analysis is 2 to 3 days. In making investigations of the effect of acids or various leather-making materials on leather, it is also necessary to wait a considerable length of time before any significant results are obtained.

In order to simplify the present testing methods and to make investigations on a more rapid scale, Joseph R. Kanagy of the Bureau's leather section has started a study of the behavior of leather in the oxygen bomb as preliminary work in the development of an accelerated aging test. As described in the *Journal of Research* for June (RP1004), chestnut and quebracho-tanned leathers treated so as to have pH values ranging from 2 to 5, were exposed in the oxygen bomb under various conditions. Quebracho leather showed the greater stability under all conditions. An explanation for this difference in stability is suggested by a consideration of the generally accepted structures of these two tannins. Comparative data on the stability of various leathers under

these conditions indicate that chrome leather is most stable, followed in order of decreasing stability by quebracho, a commercial blend, and chestnut. All results parallel those obtained previously under natural aging conditions.

PRESERVATION OF RECORDS.

The results of further studies made as a part of the Bureau's effort to obtain comprehensive information on the preservation of records, are summarized in Miscellaneous Publication M154, a revised edition of the general report of this research.

Because of the rapidly growing use of motion picture films as a means of reproducing record material, either as a preservative measure or for more general accessibility, the research was extended to a study of the stability and optimum storage conditions of films. Both the acetate and the nitrate films were studied. The former was found to be a stable material, but information on the resistance of the emulsion to wear and other influences is incomplete. The nitrate film showed a comparatively short life, but information was developed on the most suitable conditions for maximum longevity in storage and use of both this and the acetate type. The study was made possible by the financial support of the Carnegie Foundation and the National Archives. The latter also supported an investigation to determine whether certain fumigants could be used to rid documentary material of insect life without injuring the material, because all incoming material is treated in fumigating chambers before it goes to the storage stacks. All of the fumigants studied were found to be harmless to paper.

Some revision has also been made in the summaries of the previous studies. These studies were initiated by a survey of the relation of conditions of storage in libraries to the condition of stored documentary material. The results indicate that light, temperature, and the humidity and purity of the air, are conditions which need careful control to minimize deterioration of stored material. Air acidified by sulphur dioxide from the combustion of fuels was found to be

deteriorative to material stored in libraries of congested areas, but it was proved by investigation in the Folger Shakespeare Library that library air can be completely cleansed of this deteriorative gas. As a result of these studies, it is recommended that good library practice should include the use of diffused illumination, and as little of it as feasible, and the use of an air-conditioning system for the maintenance of moderate temperature and humidity and the removal of acidic gases from the air.

Tests of hundreds of old and modern papers showed the necessity for careful choice of paper for permanent record use, and that the selection should be based on the purity of the fiber, rather than its source, and on the purity of the paper in other respects. Protective coatings for papers were also given attention, and both transparent cellulose sheeting and Japanese tissue papers were found suitable.

Writing ink was known to be more or less deteriorative to paper. More exact information on this was obtained, which showed that the ordinary type of ink is distinctly injurious to all types of record papers. But the results of study of a special type of ink proposed by other investigators indicated that it might be used without harmful effect.

The world-wide interest in these studies indicates that posterity will not be cheated of its rightful heritage of the accumulated recorded knowledge of preceding generations. There are many evidences that improved library practices are being inaugurated as the result of this work.

Copies of the new publication are obtainable from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 10 cents each.

The Value of Octane Numbers in the Air.—D. P. BARNARD (*National Petroleum News*, Vol. XXIX, No. 13). Speaking before the National Aeronautic Meeting of the Society of Automotive Engineers, Mr. Barnard stated that the development of new super gasolines of high octane rating will enable commercial airplanes to carry larger payloads, reach higher speeds and increase the cruising distance when the engines are designed for this fuel. Depending upon type of vehicle and conditions of operation, the value of an improvement of one octane number may range from slightly over two cents to nearly eight cents per gallon. For moderate octane number increases, raising compression ratios seems to offer a slight advantage over supercharging. Above 100 octane number, a distinct advantage for increasing supercharge is indicated. Decreases in energy content of the fuel result in earning capacity losses equivalent to about two octane numbers per one per cent. reduction in heating value. These conclusions apply only when the possible advantages are allowed for in design, for the earning power of an existing airplane cannot be increased materially by simply increasing the octane number of the fuel.

R. H. O.

THE FRANKLIN INSTITUTE.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, June 9, 1937.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JUNE 9, 1937.

DR. WINTHROP R. WRIGHT *in the Chair.*

The following report was presented for final action:

No. 3024: Z. D. System of Monolithic Concrete Construction.

This report recommended the award of an Edward Longstreth Medal jointly to Carl Zeiss, Inc., of Jena, and the Dyckerhoff and Widmann Aktien Gesellschaft of Berlin, "In consideration of their conception and development of a new concrete shell type of roof construction." Special and particular commendation and mention were made of Walther Bauersfeld, Franz Dischinger, Ulrich Finsterwalder, Hubert Rüsch, and W. Flügge, "who contributed to the development which made the structures actual and successful creations."

JOHN FRAZER,
Secretary to the Committee.

LIBRARY NOTES.

The library is open for consultation daily from 9 o'clock A.M. until 5 o'clock P.M. and Saturdays 9 A.M. until noon. 675 readers made use of its facilities during the twenty-five days of May.

RECENT ADDITIONS.

Agriculture.

Royal Agricultural Society of England. Journal. Volume 97. 1936.

Bibliography.

Engineering Index 1936. 1937.
New York Public Library. Bulletin. Index to Volumes 1–40, 1897–1936. 1937.
SCHMECKEBIER, LAURENCE F. Government Publications and Their Use. 1936.

Chemistry and Chemical Technology.

ABEGG, R., FR. AUERBACH, UND I. KOPPEL. Handbuch der anorganischen Chemie. Vierter Band, Dritte Abteilung, vierter Teil, Lieferung 1. 1937.
American Gas Association. Proceedings. Volume 18, 1936.
Chemical Society. Annual Reports on the Progress of Chemistry for 1936. Volume 33. 1937.
Chemisches Zentralblatt. General register VIII über die Jahrgänge 1930–1934. Teil 3: Formelregister. 1937.

HOPKINS, R. H., AND B. KRAUSE. Biochemistry Applied to Malting and Brewing. 1937.

TAUBER, HENRY. Enzyme Chemistry. 1937.

Electricity.

HALL, GEORGE H. Motor and Control Applications. First Edition. 1937.

MICHAEL, WALDEMAR. Theorie der Wechselstrommaschinen in vektorieller Darstellung. 1937.

Forgery.

RHODES, HENRY T. F. The Craft of Forgery. 1937.

Founding.

American Foundrymen's Association. Transactions. Volume 44. 1936.

Medicine.

PARDEE, HAROLD E. B. Clinical Aspects of the Electrocardiogram Including the Cardiac Arrythmias. Third Edition, Revised. 1933.

Metallurgy.

BASSETT, HENRY NORMAN. Bearing Metals and Alloys. 1937.

Naval Architecture and Navigation.

Brassey's Naval Annual 1937. 48th Year of Publication.

Physics.

DODGE, RUSSELL A., AND MILTON J. THOMPSON. Fluid Mechanics. First Edition. 1937.

HAAS, ARTHUR. Elementare Physik. 1937.

Psychology.

RHINE, J. B. Extra-Sensory Perception. 1935.

Refrigeration.

MACINTIRE, H. J. Refrigeration Engineering. 1937.

BOOK REVIEWS.

ELEMENTARE PHYSIK, von Arthur Haas. 204 pages, illustrations, 14 × 21 cms. 4.80 marks, Wien, Julius Springer, 1937.

There is no claim laid to this little book as being of value as a text, rather is it a feature for its compactness and broad coverage. Only a glance through its some two hundred pages reveals an unbelievable amount of detailed knowledge. Yet, its size is such that it may be slipped into one's pocket.

The book is divided into four parts—mechanics and acoustics, heat, optics, and electricity. The most important fundamentals of each of these are discussed, sufficient for the basis for solution of problems in these fields. Because of the limited size and character of the work, the treatment makes reference to and gives examples of physical deductions, with the use of mathematics only where necessary. There is a name and subject index, which contributes much to the value of the book. For handy, brief reference on the high points of these fields, this little book should find much use.

R. H. OPPERMANN.

THEORIE DER WECHSELSTROMMASCHINEN IN VEKTORIELLER DARSTELLUNG, von Dr. Waldemar Michael. 272 pages, illustrations, 16 × 25 cms., 16.50 marks RM, Leipzig, B. G. Teubner, 1937.

The author of this book stresses the value of the use of vectors in calculations in alternating current machinery. Not only is this method a means to an end, but what is more important, it serves to facilitate understanding as to the theory. Accordingly, the author begins his work with a chapter on calculations using complex vector notation. Many, of course, can assume knowledge of this and skip it entirely. The second chapter is devoted to the fundamentals of physics, upon which the rest of the work is based. This chapter covers Ohm's law and Kirchoff's law, the magnetic field, electromagnetic induction, single and polyphase systems, etc. Both of these chapters, while elementary, cover their subjects exceedingly well and in view of this, and in order to become acquainted with the author's method of presentation and attention to details, a word of caution is necessary and the student should not pass over them too quickly.

The following chapters cover in order the transformer, the synchronous machine, the polyphase induction motor and the single phase machine. After this, appears what may be termed the second part of the book. It starts with the derivation of fundamental conceptions of alternating currents in wound rotors and gives an exceptionally clear treatment on commutation, in-so-far as is necessary to form a basis for the rest of the work. Wound rotor motors are treated in detail, also inquiry is made to gain acquaintance with the single phase shunt motor. It is stated that the theory of the latter was until now little developed. The last two chapters of the book treat on auxiliary and miscellaneous machinery.

There is an extensive bibliography for each of the chapters, at the end of the book, as well as a subject index. The approach to each point made shows a carefully thought-out procedure and the book is a text that is certainly worth while.

R. H. OPPERMANN.

MATTHEW BOULTON, by H. W. Dickinson. 218 pages, plates, illustrations, 16 × 25 cms., Cambridge University Press, 1937. $4.50.

The name of Matthew Boulton is not always associated with that of James Watt, in fact it may not be going too far to say that generally little is known of Matthew Boulton. Yet, without the association of Watt with Boulton the outcome of Watt's work would have been doubtful. Boulton furnished enthusiasm and enterprise, and he created the environment in which Watt was enabled to work out those further improvements in the steam engine that transformed it from an apparatus merely for lifting water, into one with an immensely

larger field of application to general power purposes. These improvements were so great that they amounted to a new birth of the engine.

This book is a biography of Matthew Boulton. The story reveals that Boulton was a man of many interests, aside from the steam engine, and his contributions to mankind were varied and great. He was the most prominent citizen of Birmingham during the eighteenth century, a period of unexampled development; his public-spirited actions did much to raise the status of the town by helping to remove the stigma under which its products labored; he brought in new industries and enlarged old ones; and he was largely responsible for the introduction of the new system of industry known as the factory system. Many of his productions were of high artistic merit and are only now beginning to be recognized and appreciated at their true worth.

The book is a remarkable work. It fills a gap in biographical literature that needed filling. The presentation is interesting, impressive, and authoritative.

R. H. OPPERMANN.

THE WORLD OF ATOMS, Twelve Non-Mathematical Lectures, by Arthur Haas. Second edition, enlarged and revised, translated by George B. Welch. 183 pages, illustrations, 16 × 24 cms. New York, D. Van Nostrand Company, Inc., 1937. Price $3.00.

This is the second edition of a book containing twelve lectures which were originally given before audiences comprising all faculties at the University of Vienna in 1926. The obvious reason for the second edition is to bring the subject matter up to date by an inclusion of new material and a thorough revision of the old.

The lectures have the following titles: matter and electricity; the building stones of the atoms; light quanta; spectra and energy levels; the elements; the atom as a planetary system; molecules; radioactivity; methods of atomic disintegration; the results of atomic disintegration; cosmic rays, the positron, and induced radioactivity; wave mechanics of the atom. From this list it can be seen that quite a good coverage is given of the subject. Examination of the book contents reveals an omission of all superfluous material, and a well planned and organized treatment. In this sense the book has a popular coverage, a broad, not too detailed account of the present state of atomic physics.

R. H. OPPERMANN.

ATOMIC SPECTRA AND ATOMIC STRUCTURE, by Gerhard Herzberg, translated with the co-operation of the author by J. W. T. Spinks. 257 pages, illustrations, 15 × 23 cms. New York, Prentice-Hall, Inc., 1937. Price $4.25.

In recent years the physical world has witnessed a number of changes that have far-reaching importance. Experiments are in progress today having promise of untold possibilities. All this has caused an impetus to be given to the subject of atomic physics and interest is ever increasing. There is little wonder that texts on the subject are coming to the fore, particularly those on fundamentals or introductory to basic theories.

This book is the translation of a volume published in German by Theodor Steinkopff (G. Herzberg, "Atomspektren und Atomstruktur," Dresden 1936). It is an elementary introduction, and according to the author, especially adapted to

the beginner in this field and also to those who require a certain knowledge of the subject because of its applications in other fields.

The treatment is concerned exclusively with optical line spectra in the restricted sense of the term—that is, with atomic spectra in the region from 40Å to the far infrared, and not with X-ray spectra. After a general explanatory introduction the book is divided into six main divisions. It begins with the simplest line spectra and the elements of atomic theory which includes passages on the Bohr theory of Balmer terms and wave mechanics or quantum mechanics. Then there follows multiplet structure of line spectra and electron spin, the building-up principle and the periodic system of the elements, and then a division devoted to the finer details of atomic spectra. Nuclear spin is discussed in the division on hyperfine structure of spectral lines in cases where the isotope effect does not give sufficient explanation—when the number of hyperfine structure components is greater than the number of isotopes. In this section there is included a table of observed values for nuclear spin. The table is complete for results published up to the beginning of the present year. Such completeness is not attempted anywhere in the book, with one exception, as it will be remembered that stress is laid primarily on basic principle only. The exception is another table which is on ionization potentials of the elements, in the last division of the book wherein some experimental results and applications are discussed.

For its size, the book covers much territory. Experimental results are the starting points of the theoretical considerations and while mathematics is necessarily bound very closely to the subject, complicated developments have been avoided as much as possible to be consistent with the aims of the book. A bibliography is appended containing sources of such information.

The work is well done and should be given serious consideration by those for whom it is intended.

R. H. OPPERMANN.

PASCAL, THE LIFE OF GENIUS, by Morris Bishop. 398 pages, illustrations, plates, 16 × 24 cms. Baltimore, The Williams and Wilkins Company, 1936. Price $3.50.

Everyone who has studied Physics knows of Pascal's law. But aside from brief references, little is generally known of the man. That he was a genius, this book reveals from the very beginning. Blaise Pascal was born in Clermont, France, June 19, 1623, the son of a government official who was an imperious father, sacrificing himself for his children.

When Blaise was but three years of age he already surprised his father by his questions on the nature of things and in the years immediately following took a great liking to geometry. The boy exhibited so much promise in this direction that his father introduced him to the members of the Academie libre which met every Thursday to discuss scientific matters, particularly mathematics and physics. This was when Blaise was thirteen years of age. At sixteen he wrote a treatise on conic sections which, it is stated, is the herald of modern projective geometry. At nineteen he invented, constructed, and offered for sale the first calculating machine. He gave the well known Pascal's law to physics, proved the existance of the vacuum, and helped to establish the science of hydrodynamics. In a discussion of the division of a gambler's stakes, he created the mathematical

theory of probability. His speculations were important in the early development of the infinitesimal calculus.

At the age of thirty-one, Pascal abandoned science, returning to it only to solve the problems of the cycloid. Espousing the theological principles of the Jansenists, he wrote, in their defense the Lettres provinciales which are still of interest. As a writer, his simplicity of style, determined the shape and character of the French literary language. Still more of his accomplishments were a new method of teaching reading and the organization of the first omnibus line. Pascal died at the age of 39. Truly, he was one of the world's greatest.

Biographies of great men catch the interest of many. The author of this book has assembled a large collection of verifiable information, made a thorough study of it and used unusual skill in writing the story. The book is one which will contribute much to culture, especially for those following technical professions.

R. H. OPPERMANN.

ATOMIC SPECTRA, AND THE VECTOR MODEL, by A. C. Candler. Volume I, Series Spectra; Volume II, Complex Spectra. Two volumes, illustrations, diagrams, 14 × 22 cms. Cambridge, the University Press, 1937. Price $8.50.

This is a set of two volumes that can best be covered in the author's own words.

"The empirical laws on which modern spectroscopy is based were all worked out with a notation which seems to-day cumbrous and misleading; yet since 1929, when a large group of physicists adopted the modern simplifications as standard, no adequate review of the earlier work seems to have appeared in this country; the beginner must still refer to Fowler's Report and master a notation which he will soon wish to forget. To repair this omission is the chief aim of the first volume. In addition, an account is given of the splitting of spectral lines in a magnetic field, since this is essential to a full understanding of the series laws.

"As Fowler's Report ordered series spectra, so Hund's Linienspektren ordered the spectra of the elements and related them to the periodic system; if the evidence was impressive in 1927, it is now overwhelming; the theory is developed and the spectral types described in five chapters of the second volume.

"Besides these two chief topics, there are others which are more or less closely related; some of these, notably hyperfine structure, are of very recent growth. The last chapters take up these topics one by one, and lead the reader to the boundaries on which research is now concentrated.

"In the hope that the argument may remain within the easy comprehension of every working physicist, the vector model has been used throughout. Where the quantum mechanics has refined the predictions of the model, the results have been freely quoted and compared with experiment; but the rather heavy mathematics, which is the basis of even the simplest calculation, has been wholly excluded."

The set should satisfy a very pressing need not only for the beginner as a time saver but as a reference for the physicist in the light of a translation to modern notation.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

The Newer Alchemy, by Lord Rutherford. 67 pages, illustrations, plates, 12.5 × 19 cms. New York, The Macmillan Company, 1937. Price $1.50.

Electricity and Magnetism, an Introduction to the Mathematical Theory, by A. S. Ramsey. 267 pages, illustrations, 14 × 22 cms. Cambridge University Press, 1937. Price $3.25.

Marconi, The Man and His Wireless, by Orrin E. Dunlap, Jr. 360 pages, plates, 15 × 22 cms. New York, The Macmillan Company, 1937. Price $3.50.

Atomic Structure of Minerals, by W. L. Bragg. 282 pages, illustrations, plates, 15 × 23 cms. Ithaca, New York, Cornell University Press, 1937. Price $3.75.

Man in a Chemical World, the Service of Chemical Industry, by A. Cressy Morrison. 292 pages, illustrations, 16 × 22.5 cms. New York, Charles Scribner's Sons, 1937. Price $3.00.

The Reactions of Pure Hydrocarbons, by Gustav Egloff. 897 pages, illustrations, 15.5 × 23.5 cms. New York, Reinhold Publishing Corporation, 1937. Price $16.75.

Technology and the Mineral Industries, by F. G. Tryon, K. C. Heald, T. T. Read, G. S. Rice, and Oliver Bowles. Prepared for the National Resources Committee. Report on Technological Trends and Their Social Implications. Report No. E-1. 63 pages, illustrations, 17.5 × 25.5 cms. Philadelphia, Pennsylvania, 1937.

National Advisory Committee for Aeronautics. Aircraft Circulars: No. 206, Armstrong Whitworth 27 "Ensign" Commercial Airplane (British), An All-Metal High-Wing Monoplane. 8 pages, illustrations. Technical Notes: No. 601, The Lateral Instability of Deep Rectangular Beams, by C. Dumont and H. N. Hill, Aluminum Company of America. 28 pages, illustrations, tables. No. 602, Heat Transfer from Cylinders Having Closely Spaced Fins, by Arnold E. Biermann, Langley Memorial Aeronautical Laboratory. 11 pages, illustrations. 3 pamphlets, 20 × 26 cms. Washington, Committee, 1937.

CURRENT TOPICS.

Submarine Power.—(*Power*, Vol. 81, No. 4.) The hydro plant at Rostin, Pomerania, Germany, which went into operation January 17 is said to be the world's first under water power station. The plant is built beneath the dam and must be entered through a manhole at one end. Two small turbines attended by one man supply all neighboring villages with cheap power. The cost is claimed to be much less than that of a conventional station.

R. H. O.

The World's Highest Chimney.—(*Power Plant Engineering*, Vol. XLI, No. 4.) Curiously, the world's highest chimney is in Korea, and it is not part of a power plant but a smelter. It was built for the Nippon Mining Co. at Chinnampu, Korea, under the Weber system of reinforced chimney construction and with Weber unit forms. The chimney is 618 feet from the bottom of the foundation to the top. The inside diameter at the top is 25 ft. The thickness of the shaft at the foundation is 37 in. and at the top 7 in. It has a lining which is 6 in. thick, reinforced both vertically and horizontally, and is 180 ft. high. There is an air space of 4 in. between the lining and the outer shell, or shaft. The foundation is circular, with a diameter of 97 ft. and contains 3200 cu. yds. of concrete. There are 4400 cu. yds. of concrete in the shaft and it contains 320 tons of steel bars, with 120 tons in the foundation. The foundation is 18 ft. thick.

R. H. O.

Hydraulic Cartridge Fractures Mass Masonry.—(*Construction Methods and Equipment*, Vol. 19, No. 3.) Cartridges inserted in drilled holes have for the last 7 years broken concrete and other types of mass masonry into large blocks for removal from demolition jobs of the Hydraulic Concrete Breaking Co. of Detroit. The method utilizes an hydraulic cartridge of 90 lb. weight consisting of a strong steel cylinder 4 in. in diameter with rams on one side which are thrust out by hydraulic pressure transmitted through lead pipe from a small pressure-generating chamber. A pressure of 20 tons per square inch can be exerted by the cartridge. Because it works without shock or creation of dust, the hydraulic cartridge can be used where explosives cannot. Although the hydraulic method of

demolishing concrete is more expensive than explosives, it was found that the ability to break blocks of 2 to 10 tons for loading into trucks or railroad cars often means a saving in final cost as well as greater speed in completing the work.

R. H. O.

X-Raying for Safety.—(*Safety Engineering*, Vol. 73, No. 3.) Modern industrial safety engineering is vitally concerned in preventing injuries that may take many years to incapacitate the workman and sometimes take a long time even to make their presence manifest to the workman. The doctor now has a tool of his own to detect the oncoming trouble before it reaches serious proportions. This is the x-ray machine. Previously the cost of such a machine for this use has been a very important item. Now there seems to be a method and a medium which offers a solution of the cost problem particularly in chest radiographs. This recent development is receiving wide recognition and being applied to large groups of individuals for the purpose of screening out cases requiring attention or further examination. The method uses paper film in rolls and comprises a series of coordinated procedures and processes which so speeds up the marking, making, processing, reading and recording of diagnoses, that a thousand or more chest radiographs may be made in an 8 hour day.

R. H. O.

Strength of Materials.—A Special Summer Program and Conferences on Strength of Materials will be held at the Massachusetts Institute of Technology for four weeks beginning June 21, 1937. There will be lectures on creep, fatigue of metals, and strength of materials by prominent men in these fields including the staff of the Institute. Laboratory exercises will be given in the testing of metals, making use of the more modern measuring instruments and apparatus. Four Seminars will be held during the course, to afford opportunity for the presentation of recent developments in allied fields of engineering mechanics.

R. H. O.

Radio Device Locates Underground Pipe Lines.—(*American Gas Journal*, Vol. 146, No. 4.) A new instrument known as the M-Scope, an invention of Dr. Gerhart R. Fisher, Director of the Fisher Research Laboratories, is now being used successfully for locating and mapping gas distribution lines. It consists essentially of two specially designed radio units—a transmitter and receiver—the former sending out a continuous buzzing signal. When there is

no metal interposed between the units, a definite volume of sound is heard in the earphones of the receiver where a meter also gives a definite reading. If a metal object, even deeply buried, lies between the units, the volume of the signal increases and the meter reading becomes higher. In using the M-Scope for tracing an underground pipe line, the transmitter is set on the ground directly above it, and the operator, carrying the receiver, follows the path along which sustained signals are received. Sudden increases in the signals mean additional metal—a valve, junction, etc., just below. Sudden decreases mean that the line has turned aside or ended. Energy for both units is supplied by standard dry cells which last several months in normal service. Individual units weigh about 8 lbs. each.

R. H. O.

New Transatlantic Antennas.—Recently, engineers of the huge RCA international radio communications station at Riverhead, Long Island, set a number of 130 foot wooden poles for the support of new antennas. The rigging crew of the station towed the enormous sticks to position, up-ended them with gin poles and lowered them into concrete sockets in the ground with amazing speed. The poles are believed to be the largest one-piece masts in the East, with an average length of 130 feet and an average weight of four tons. They are of Douglas fir from the state of Washington. The new antennas, when completed, will have an increased average height of 60 to 70 per cent. over those previously employed, which is expected to result in more efficient operation in the longer part of the so-called short wave band of radio wave-lengths.

R. H. O.

If Lumber Warps, Maybe It's "Compression Wood."—Anyone who handles lumber from coniferous trees is familiar with boards that bow, twist, and warp. The cause may be compression wood in the board, says a bulletin just issued by the Forrest Products Laboratory of the U. S. Dept. of Agriculture at Madison, Wis. Compression wood troubles usually develop in lumber from the lower side of a large branch or lower side of a leaning tree trunk. Pressure from the weight of the branch or the top of the leaning tree deforms the individual cells. When the lumber is sawed and the pressure removed, these abnormal cells begin to misbehave with every change in moisture. The wood fibers on opposite sides of a board do not shrink and swell in the same way and in the same proportion. Such lumber bends and twists and pulls out of shape. Compression wood is low in strength for its weight and

hard to nail or work with tools. It can be identified in logs by the eccentric annual growth rings.

R. H. O.

Fulham Washes Flue Gas.—D. W. RUDORFF. (*Power*, Vol. 81, No. 6.) The new Fulham power plant is in the midst of a thickly populated part of London, England. This 300,000 k.w. plant will, when it is completed, consume daily 2,200 tons of coal containing from 0.75 to 1.5 per cent. of sulphur. In view of this it was considered very necessary to remove the sulphur from the stack discharge. After research was undertaken, there finally evolved a washing process known as the Howden ICI system of gas scrubbing. This is a closed liquor process, so that it is unnecessary to discharge waste products into the Thames River, a very important point. The washing plant consists of a scrubber system in which intimate contact between washing liquid and flue gas is obtained by a combination of rain and surface film action. To achieve a substantially complete absorption and neutralization of the SO_2 and other acid constituents, milk of lime is added to the water. This lime combines with CO_2 absorbed from the flue gas to produce both chalk (which is insoluble) and calcium carbonate (which is soluble). This latter constitutes the active absorbing and neutralizing agent, and fresh amounts are continually formed from chalk as the absorption and neutralization of the acid constituents proceed. To keep up the supply of calcium bicarbonate and to prevent the recirculating liquor from becoming acid, fresh milk of lime is fed into the system. SO_2 and SO_3 contained in the flue gas are absorbed as calcium sulphite and calcium sulphate, and the washing liquor becomes charged with these relatively insoluble compounds. As soon as the respective saturation points are exceeded, the salts are precipitated. This occurs, not within the scrubber, but in a part of the recirculating system chosen and especially designed for this purpose.

R. H. O.

Fluid Flow Analyzer.—H. L. PARR. (*Mechanical Engineering*, Vol. 59, No. 6.) Research work in fluid dynamics, carried on in the last few years chiefly for the benefit of airplane designers, has shown engineers, engaged in many other fields, that a knowledge of some fluid flow characteristics is necessary for intelligent design of a wide range apparatus. Heretofore apparatus has been built with which such information can be obtained, but it has been somewhat expensive. An inexpensive device that is simple to operate has

been designed and built by the author. The main feature of this apparatus is that the flow takes place between two flat surfaces set so close together that considerable air velocities can be used, and little protection is necessary to maintain parallel flow. A box $48 \times 7 \times 12\frac{1}{2}$ inches contains the apparatus. Two plates of glass are fitted horizontally on this box so that there is a $\frac{1}{4}$ inch deep and 12 inch wide channel between them. This channel connects the inlet and outlet compartments of the box at the ends. Arrangement is made so that smoke from titanium tetrachloride flows in the channel between the glass plates. Rubber models of various shapes and sizes, cut to the thickness of the channel are placed in the channel and streamlines can be observed around them. This is best accomplished by lighting from beneath the glass plates. Satisfactory photographs have been taken by a camera with illumination from either the top or the bottom. While the results obtained by such an apparatus are largely qualitative, they have a wide practical application in addition to their usefulness in illustrating the principles of fluid flow.

R. H. O.

Vehicle Tunnel to Be Built Under Maas River at Rotterdam. —(*Engineering News Record*, Vol. 118, No. 21.) Plans for the construction of this tunnel which separates two sections of the city of Rotterdam, Holland, call for a tunnel 2,295 ft. long exclusive of shore approaches. It is to be built by sinking ten preconstructed sections in line and joining their ends under water and will have a capacity of 5,200,000 vehicles a year besides bicycle and pedestrian traffic. Construction, it is expected, will be completed by January 1941.

R. H. O.